Magnetic Field Calculation Of Square Coils Having Rounded Corners

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***Abstract*—There are differently shaped coils for induction heating used in industry such as circular, and square coils. While producing square-like shaped coils, it is unavoidable to face production difficulties due to the sharp corners. At the end of the production, instead of square-like shape, edges are flat and corners are rounded shape coil which is called squircle coil is produced. This paper will discuss how these rounded corners effect the magnetic field at the center of the coil by deriving missed formulas in the literature, and prove the results by comparing with the results of generally known formulas.**

Keywords—Squircle shaped coils, magnetic flux density calculation

I. Introduction

Heating occupies an important place in industry. By heating a material, it is easy to form it. In addition, heating might be necessary for a material's state changes such as boiling a water. With the development of technology, in today's world there are different kinds of heating methods that are being widely used including resistive and infrared heating, etc. Among these methods, induction heating is increasingly becoming more popular because it is clean, efficient, cost-effective, precise, and controllable method. Induction heating is based on creating eddy currents on a conducting material that is placed in and exposed to a varying magnetic field. One of the most important parts of induction systems is coil. Coil creates magnetic field when electrical power is applied on and current flows through it.

In industry it is possible to see various coil architectures such as circular and square shaped coils [1]-[3]. Depending on application, coil structure plays an important role on system efficiency. For instance, in conventional hobs to heat circular shaped pans with a single coil, circular shaped coils are preferred [4]. On the other hand, in all surface induction hobs coils having geometries with long tangential sides that enable high coupling are mostly used. However, because of their sharp corners, it is not easy to produce coils having square type shapes. During manufacturing, wires in sharp corners of square shaped coils are rounded. This affects the magnetic field created by the coil. The amount of change in the produced field depends on the points at which rounding starts. Also, this change is not the same at every point.

Rounding of coil changes the magnetic field created at the center of the coil. Amount of this change depends on coil sizes. Squircle coils were examined before experimentally in all-surface induction heating systems. However, there is no report in literature analytically investigating change of produced magnetic field and other parameters of a coil with rounding of its corners. To this end, here we examined analytically change of produced magnetic field at the center of a square shaped coil with rounding its corners, i.e., while square coil is evolved into a squircle coil, for different arc and edge lengths.

II. Structure and Results

A. Structure

Planar coils used in conventional induction systems have square and circular geometries. Square and circular shaped coils are simply shown in Fig. 1.

Fig. 1. Conventional coil structures: a) circular shaped coil and b) square shaped coil.

Other than conventional circular and square coil shapes, squircle shaped coil geometries, squircle-shaped coils are also used for induction heating [5]. Squircle coil geometry is illustrated in Fig.2.

Fig. 2. Squircle coil architecture.

A square shaped coil is evolved into a squircle coil as its sharp corners are rounded (see Fig.1 and Fig.2). Depending on the positions of the points where rounding starts and ends, squircle geometry gets closer to either a square geometry or a circular geometry. In other words, a circular coil can be thought as a special form of a squircle shaped coil on which rounding starts at the middle point of sides and straight parts on the sides have zero length.

B. Analyticial Derivations

We began our analysis by calculating magnetic flux density contributed by an arc at the corner of the squircle coil. For that, we first obtained geometrical derivations. In Fig.3, part of a squircle shaped coil geometry is illustrated.

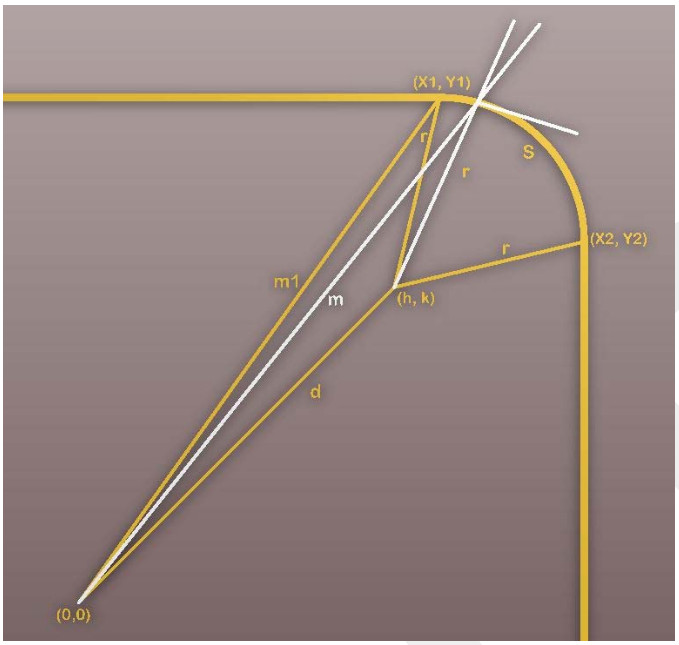

Fig. 3. Part of a squircle shape together with geometrical parameters.

In the analysis it is assumed that total length of wire, length of straight sides, and length of arcs are known together with start and end points of arcs. In the figure, center of the squircle geometry is selected to coincide with the (0,0) point. Also, start and end points of the arc is pointed out with (X_1, Y_1) and (X_2, Y_2) coordinates, respectively. On the other hand, in the figure, m1 represents the distance between the center of the squircle geometry and start point of the arc. Similarly, m is the distance between the center of the squircle geometry and an arbitrary point on the arc. In the figure, arc center is marked with (h, k) coordinates and the distance between center of squircle geometry and arc center is named as d. Moreover, radius and length of the arc are labeled as r and S, respectively, in the figure.

Fig.4 shows closer view of part of the squircle coil geometry.

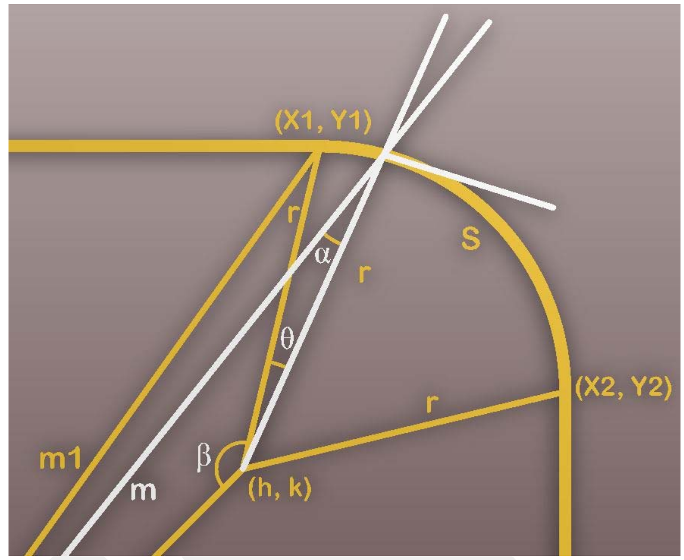

Fig. 4. Closer view of part of a squircle shape together with length parameters and angles.

In our derivations, we initially calculated the distance m1 between the center of the squircle geometry and start point of the arc (X_1, Y_1). The relation is given in (1)

$$m_1=\sqrt{x_1^2+y_1^2} \quad (1)$$

By using triangle rule, we can define m1 as functions of arc radius(r) and the distance between center of squircle geometry and arc center(d), together with angle between d and line that passes from points (h, k) and (X, Y). The relation is written in (2), where cosine of β angle is left alone

$$\cos(\beta)=\frac{r^2+d^2-m_1^2}{2rd} \quad (2)$$

Similarly, relation between parameters (β+θ), r, d, and m in Fig.4 can be found as in (3)

$$\cos(\beta+\theta)=\frac{r^2+d^2-m^2}{2rd} \quad (3)$$

The cosine term in (3) is expanded as functions of sines and cosines of angles β and θ. Cosine of angle β is formulated in (2)

$$\sin(\beta)=\sqrt{1-\cos^2(\beta)} \quad (4)$$

$$\sin(\beta)=\sqrt{1-\left[\frac{r^2+d^2-m_1^2}{2rd}\right]^2} \quad (5)$$

$$\cos(\beta+\theta)=\cos(\beta).\cos(\theta)-\sin(\beta).\sin(\theta) \quad (6)$$

(4) is a generally known trigonometrical formula, and by placing (2) into (4), we can achieve (5). Similarly, (6) is another trigonometrical formula and if we place (3), (2), and (5) in to (6), we derive (7).

$$\frac{r^2+d^2-m^2}{2rd}=\frac{r^2+d^2-m_1^2}{2rd}\cos(\theta)-\sqrt{1-\left[\frac{r^2+d^2-m_1^2}{2rd}\right]^2}\sin(\theta) \quad (7)$$

By leaving m alone in (7), we have (8).

$$m=\sqrt{-(r^2+d^2-m_1^2)\cos(f)+2rd\sqrt{1-\left[\frac{r^2+d^2-m_1^2}{2rd}\right]^2}\sin(f)+r^2+d^2} \quad (8)$$

$$d^2=m^2+r^2-2mr\cos(\alpha) \quad (9)$$

(9) is generally known triangle rule which helps us to determine cos equation of angle α which is (10).

$$\cos(\alpha)=\frac{m^2+r^2-d^2}{2mr} \quad (10)$$

In (11), traditional biot-savart equation is shown.

$$B=\frac{\mu_0}{4\pi}\oint_c I\frac{dlxr}{r^2} \quad (11)$$

If biot-savart rule is applied to our case, and if central angle of the arc is called as "θ_1" we have (12).

$$B=\frac{u_0 I}{4\pi}\int_0^{\theta_1}\frac{r\cos(\alpha)}{m^2}d\theta \quad (12)$$

If (8) placed in to the (12) we get (13).

$$B=\frac{u_0 I}{8\pi}\int_0^{\theta_1}\frac{m^2+r^2-d^2}{m^3}d\theta \quad (13)$$

It is not easy to obtain a solution for magnetic flux density B in (13). To get a result, we have used Simpson's rule. We reported our final solution for different arc lengths.

Squircle coils compose of rounded corners and straight edges. After calculating magnetic flux density created by the corners via (13), we found magnetic field created by the edges simply by applying straight wire's magnetic field formula [6].

C. Results

Magnetic flux density at center points of squircle coils were calculated for various coil sizes. For fair comparisons, side lengths of the squircle coils were selected to be the same with diameters of some of commercially available vessels. Calculations were made for side lengths of 7.5 cm, 16.0 cm, and 28.0 cm. Side lengths of 7.5 cm and 28.0 cm corresponds to diameters of a coffee pot and a big utensil daily used by consumers in homes. In addition, for each squircle coil with a constant side length calculation were repeated for different arc lengths of straight edges. In other words, calculations were repeated for different start and end points of rounding on a squircle geometry whose side lengths are set to be constant. In our analysis, symmetry of a squircle coil is hold, i.e., all the sides in four directions are the same. Magnetic flux densities found with our derived formula are given in Table I, Table II, and Table III for side lengths of 7.5 cm, 16.0 cm, and 28.0 cm, respectively, together with the magnetic field densities calculated with known formulas of straight lines and a circular coil. In the table percentage error is also given.

For each different size of coil, different length of edge and arc sub-cases are selected to see how different amount of rounding effects the magnetic field created at the middle of the coil. Addition of this, we also calculated each sub-case with the formula to calculate our error rate of the results [6]. Results are shown in Table I to Table III.

Since the proposed formulization is applicable to any squircle shaped coil, we selected squircle shapes which has arcs such that their center is at the point of (0, 0). This is preferred due to the fact that traditional formulization is only applicable to only squircle shaped coil which has arcs whose center overlaps with center of squircle shape. In this way, we have fair comparison between traditional and proposed methods.

Table I. Magnetic flux densities at the center of a squircle coil having 7.5 cm side length and percentage errors.

Arc Start and End Points	Calculated Flux Density with Proposed Formula	Calculated Flux Density with Traditional Formula	Error (%)
(3.74, 3.75) (3.75, 3.74)	0,12006210	0,12015463	0,0770
(2.5, 3.75) (3.75, 2.5)	0,12204898	0,12769304	4,4200
(1.25, 3.75) (3.75, 1.25)	0,12821097	0,13236029	3,1348
(0.01, 3.75) (3.75, 0.01)	0,13266777	0,13327647	0,4567
(0, 3.75) (3.75, 0)	0,13297590	0,142857	0,2678

Table II. Magnetic flux densities at the center of a squircle coil having 16 cm side length and percentage errors.

Arc Start and End Points	Calculated Flux Density with Proposed Formula	Calculated Flux Density with Traditional Formula	Error (%)
(7.99, 8) (8, 7.99)	0,05629124	0,05630248	0,0199
(7, 8) (8, 7)	0,05642871	0,05766849	2,1498
(5, 8) (8, 5)	0,05749736	0,06008353	4,3042
(3, 8) (8, 3)	0,05975568	0,06156119	2,9328
(0.01, 8) (8, 0.01)	0,06239952	0,06187377	0,8497
(0, 8) (8, 0)	0,0624	0,0625	0,1600

Table III. Magnetic flux densities at the center of a squircle coil having 28 cm side length and percentage errors.

Arc Start and End Points	Calculated Flux Density with Proposed Formula	Calculated Flux Density with Traditional Formula	Error (%)
(13.99, 14) – (14, 13.99)	0,03215874	0,03216540	0,0206
(11, 14) – (14, 11)	0,03236695	0,03352809	3,4631
(8, 14) – (14, 8)	0,03308266	0,03467933	4,6040
(5, 14) – (14, 5)	0,03424364	0,03541585	3,3098
(0.01, 14) – (14, 0.01)	0,03570591	0,03570583	0,0002
(0, 14) – (14, 0)	0,03571891	0,0357142	0,0131

As it can be seen in the tables, maximum error which is 4,6 percent is in the calculation of the coil which has 28 cm side length. Because it is less than 5 percent, we have proved validity of our formulization.

Also, for each of three case, we started our calculations with almost square shape case, and finish our calculations with circular shape case. In other words, in Table III, arc start and end points of (13.99, 14) – (14, 13.99) represents almost square shaped coil, and similarly arc start and end points of (0, 14) – (14, 0) represents circular coil shape. When we check the error rate of each case, we investigate that if the arc start and end points are selected such that we have squircle shape which is in the middle of square and circular shape, we get the maximum error rate.

III. Conclusion

Due to the difficulties of producing sharp corners of square shaped coils, and therefore having squircle shaped coils causes different magnetic field, different induction value, and most importantly different overall efficiency of the system. For that reason, a gap found in the literature to calculate that significantly important change in efficiency caused by rounded corners. In this study, required formulization of the magnetic field calculation is determined, and calculation are done for the minimum, medium, and maximum size of coils in the literature. Also, magnetic field value is calculated for each case by using traditional formulization to compare the results [6]. When results are compared, we saw that results are correct with as negligible as low error rates.

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