

ON THE EXISTENCE OF POSITIVE SOLUTIONS FOR THE ONE-DIMENSIONAL p -LAPLACIAN BOUNDARY VALUE PROBLEMS ON TIME SCALES

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ABSTRACT. In this paper, we study the following p -Laplacian boundary value problems on time scales

$$\begin{cases} (\phi_p(u^\Delta(t)))^\nabla + a(t)f(t, u(t), u^\Delta(t)) = 0, & t \in [0, T]_{\mathbb{T}}, \\ u(0) - B_0(u^\Delta(0)) = 0, & u^\Delta(T) = 0, \end{cases}$$

where $\phi_p(u) = |u|^{p-2}u$, for $p > 1$. We prove the existence of triple positive solutions for the one-dimensional p -Laplacian boundary value problem by using the Leggett-Williams fixed point theorem. The interesting point in this paper is that the non-linear term f is involved with first-order derivative explicitly. An example is also given to illustrate the main result.

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1. INTRODUCTION

Recently, dynamic equations on time scales have generated a considerable amount of interest and attracted many researchers. They can not only unify differential and difference equations [11], but also have exhibited much more complicated dynamics [5]. Further they have led to several important applications e.g., in the study of insect population models, stock market, wound healing, and epidemic models [6, 13, 16].

In this paper, by using different method, we will discuss the existence of at least three positive solutions to the following p -Laplacian boundary value problem on time scales

$$(1.1) \quad \begin{cases} (\phi_p(u^\Delta(t)))^\nabla + a(t)f(t, u(t), u^\Delta(t)) = 0, & t \in [0, T]_{\mathbb{T}}, \\ u(0) - B_0(u^\Delta(0)) = 0, & u^\Delta(T) = 0, \end{cases}$$

where $\phi_p(u)$ is p -Laplacian operator, i.e., $\phi_p(u) = |u|^{p-2}u$, for $p > 1$, with $(\phi_p)^{-1} = \phi_q$ and $\frac{1}{p} + \frac{1}{q} = 1$. The usual notation and terminology for time scales as can be found in [5, 6], will be used here.

Agarwal and O'Regan [2] studied the existence of one or more solutions to nonlinear equations on time scales. They established by using either a nonlinear alternative of Leray-Schauder type or Krasnoselski's fixed point theorem in a cone. Dogan et

al. [7] considered some existence criteria for positive solutions of a higher order semi-positone multi-point boundary value problem on a time scale. We also discussed applications to some special problems. He and Jiang [10] investigated the existence of at least three positive solutions of boundary value problems for p -Laplacian dynamic equations on time scales by applying a new triple fixed-point theorem. Hong [12] presented sufficient conditions for the existence of at least three positive solutions of three-point boundary value problems for p -Laplacian dynamic equations on a time scales. To show his main results, he applied a new fixed point theorem due to Avery and Peterson. Su et al. [19] considered the three-point boundary value problem for p -Laplacian dynamic equations on time scales. They proved that the boundary value problem has at least three positive pseudo-symmetric solutions under some assumptions by using a pseudo-symmetric technique and the five-functionals fixed point theorem.

In recent years, there have been many papers working on the existence of positive solutions for p -Laplacian boundary value problems for differential equations on time scales, see, for example [3, 10, 12, 15, 17, 18, 19, 20, 21]. However, to best of our knowledge, there are not much concerning the p -Laplacian boundary value problems on time scales when the nonlinear term f is involved with the first-order delta derivative [8, 9].

We assume the following conditions hold through the paper:

- (H1) $f \in C_{ld}((0, T) \times \mathbb{R}^2, (0, \infty))$, $0, T \in \mathbb{T}$,
- (H2) $a \in C_{ld}((0, T), (0, \infty))$, $\min_{t \in [0, T]_{\mathbb{T}}} a(t) = \phi_p(m)$, $\max_{t \in [0, T]_{\mathbb{T}}} a(t) = \phi_p(M)$ and $m < M$,
- (H3) B_0 is continuous function defined on \mathbb{R} and satisfies that there exist $A \geq 1$ and $B > 0$ such that $Bv \leq B_0(v) \leq Av$, for all $v \in [0, +\infty)$.

2. PRELIMINARIES

Let $E = C_{ld}^1[0, T]$ with the norm

$$\|u\| = \max\{\|u\|_0, \|u^\Delta\|_0\},$$

where $\|u\|_0 = \sup_{t \in [0, T]_{\mathbb{T}}} |u(t)|$; clearly E is Banach space. Choose the cone $P \subset E$ defined by

$$P = \left\{ u \in E : u(t) \geq 0, \text{ for } t \in [0, T]_{\mathbb{T}}; u^{\Delta \nabla}(t) \leq 0, u^\Delta(t) \geq 0, \text{ for } t \in [0, T]_{\mathbb{T}} \right\}.$$

We note that $u(t)$ is a solution of (1.1), if and only if

$$\begin{cases} u(t) = \int_0^t \phi_q \left(\int_s^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \Delta s \\ + B_0 \left(\phi_q \left(\int_0^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \right), \quad t \in [0, T]_{\mathbb{T}} \end{cases}$$

Define a completely continuous integral operator $F : P \rightarrow E$

$$\begin{cases} (Fu)(t) = \int_0^t \phi_q \left(\int_s^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \Delta s \\ + B_0 \left(\phi_q \left(\int_0^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \right), \quad u \in P \text{ for } t \in [0, T]_{\mathbb{T}}. \end{cases}$$

Clearly, $\|Fu\| = \max\{(Fu)(0), |(Fu)^\Delta(T)|\} = T_0(Fu)(0)$, where $T_0 = \max\{T, 1\}$.

Lemma 2.1. $FP \subset P$

Proof. In fact

$$(Fu)^\Delta(t) = \phi_q \left(\int_t^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \geq 0.$$

Moreover, $\phi_q(x)$ is a monotone decreasing and continuously differential function and

$$\left(\int_t^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right)^\nabla = -a(t) f(r, u(t), u^\Delta(t)) \leq 0,$$

we have $(Fu)^{\Delta\nabla}(t) \leq 0$, therefore $FP \subset P$. \square

Lemma 2.2. $F : P \rightarrow P$ is completely continuous.

Proof. Firstly, we will show that F maps a bounded set into itself. Suppose $c > 0$ is a constant and $u \in \bar{P}_c = \{u \in P : \|u\| \leq c\}$, and then $|u| \leq c$, $|v| \leq c$; notice that $f(t, u, v)$ is continuous, therefore there exist a constant $C > 0$ such that $f(t, u, v) \leq \phi_p(C)$, and hence

$$\begin{aligned} \|Fu\| &= T_0(Fu)(0) \\ &= T_0 B_0 \left(\phi_q \left(\int_0^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \right) \\ &\leq T_0 A C \phi_q \left(\int_0^T a(r) \nabla r \right). \end{aligned}$$

That is $F\bar{P}$ is uniformly bounded. On the other hand,

$$\begin{aligned} |(Fu)(t_1) - (Fu)(t_2)| &= \left| \int_0^{t_1} \phi_q \left(\int_s^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \Delta s \right. \\ &\quad \left. - \int_0^{t_2} \phi_q \left(\int_s^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \Delta s \right| \\ &\leq \left| \int_{t_1}^{t_2} \phi_q \left(\int_0^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \Delta s \right| \\ &\leq C |t_1 - t_2| \phi_q \left(\int_0^T a(r) \nabla r \right), \end{aligned}$$

therefore F is equicontinuous on $[0, T]_{\mathbb{T}}$; then by applying the Arzela-Ascoli theorem on time scales [1], we know that $F\bar{P}$ is relatively compact. Using Lebesgue's

dominated convergence theorem on time scales [4], F is completely continuous on $[0, T]_{\mathbb{T}}$. \square

Let $a, b, r > 0$ be constants, $P_r = \{u \in P : \|u\| < r\}$, $P(\alpha, a, b) = \{u \in P : \alpha(u) \geq a, \|u\| < b\}$.

To prove our main results, we need the following Leggett-William fixed point Theorem [14].

Theorem 2.3 (Leggett-Williams). *Let $F : \bar{P}_c \rightarrow \bar{P}_c$ be a completely continuous map and α be a nonnegative continuous concave functional on P such that $\alpha(u) \leq \|u\|$, $\forall u \in \bar{P}_c$. Assume there exist a, b, d with $0 < a < b < d \leq c$ such that*

- (A1) $\{u \in P(\alpha, b, d) : \alpha(u) > b\} \neq \emptyset$, and $\alpha(Fu) > b$ for all $u \in P(\alpha, b, d)$;
- (A2) $\|Fu\| < a$, for all $u \in \bar{P}_a$;
- (A3) $\alpha(Fu) > b$ for all $u \in P(\alpha, b, c)$ with $\|Fu\| > d$.

Then F has at least three fixed points u_1, u_2, u_3 satisfying

$$\|u_1\| < a, \quad b < \alpha(u_2), \quad \|u_3\| > a, \quad \alpha(u_3) < b.$$

3. MAIN RESULTS

We define the nonnegative continuous concave functional $\alpha : P \rightarrow [0, \infty)$ by

$$\alpha(u) = \min_{t \in [l, T-l]} u(t), \quad l = \max\{t \in T : t \in [0, T/2]\},$$

$$K = \left(\frac{l+B}{l}\right) \phi_q \left(\int_0^{T-l} a(r) \nabla r\right).$$

Clearly, the following two conclusions hold:

- (i) $\alpha(u) = u(l) \leq \|u\|$, $\forall u \in P$;
- (ii) $\alpha(Fu) = u(l)$.

Theorem 3.1. *Assume that there exist constants a, b, c, d such that $0 < a < b \leq \frac{(l+B)m}{MBT_0}d < d \leq c$ and suppose that f satisfies the following conditions:*

- (B1) $f(t, u, v) \leq \phi_p\left(\frac{aT^{\frac{1-p}{p}}}{MT_0}\right)$ for $(t, u, v) \in [0, T]_{\mathbb{T}} \times [0, a] \times [-a, 0]$;
- (B2) $f(t, u, v) \leq \phi_p\left(\frac{cT^{\frac{1-p}{p}}}{MT_0}\right)$ for $(t, u, v) \in [0, T]_{\mathbb{T}} \times [0, c] \times [-c, 0]$;
- (B3) $f(t, u, v) > \phi_p\left(\frac{b}{lK}\right)$ for $(t, u, v) \in [0, T-l]_{\mathbb{T}} \times [b, d] \times [-d, 0]$;
- (B4) $\min\{f(t, u, v)\} \phi_p\left(\frac{M}{m}\right) \int_0^{T-l} a(t) \nabla t \geq \max\{f(t, u, v)\} \int_0^T a(t) \nabla t$ for $(t, u, v) \in [0, T]_{\mathbb{T}} \times [0, c] \times [-c, 0]$.

Then the boundary value problem (1.1) has at least three positive solutions u_1, u_2 and u_3 satisfying

$$\|u_1\| < a, \quad b < \alpha(u_2), \quad \|u_3\| > a, \quad \alpha(u_3) < b.$$

Proof. First, we show that there exists a positive number $c > d$ such that $F\bar{P}_c \subset \bar{P}_c$, $F\bar{P}_a \subset \bar{P}_a$. From Lemma 2.1 $F\bar{P}_c \subset \bar{P}_c$, and then $\forall u \in \bar{P}_c$, from B2, we have $0 \leq u \leq c$, $-c \leq v \leq 0$,

$$\begin{aligned} \|Fu\|_0 &\leq T\phi_q \left(\int_0^T a(r)f(r, u(r), u^\Delta(r))\nabla r \right) \\ &\quad + A \left(\phi_q \left(\int_0^T a(r)f(r, u(r), u^\Delta(r))\nabla r \right) \right) \\ &= (T + A)\phi_q \left(\int_0^T a(r)f(r, u(r), u^\Delta(r))\nabla r \right) \\ &\leq (T + A)\phi_q \left(\phi_p(M)\phi_p\left(\frac{c}{T_0M}\right) \right) \leq c \\ \|(Fu)^\Delta\|_0 &\leq \phi_q \left(\phi_p(M)\phi_p\left(\frac{c}{T_0M}\right) \right) \leq c. \end{aligned}$$

Similarly, $Fu \in \bar{P}_a$ for all $u \in \bar{P}_a$.

Second, we show $\{u \in P(\alpha, b, d) : \alpha(u) > b\} \neq \emptyset$, and $\alpha(Fu) > b$ for all $u \in P(\alpha, b, d)$. In fact, set $u = \frac{b+d}{2}$, $\|u\| = \frac{b+d}{2} \leq d$ and $\alpha(u) > b$. Therefore $\{u \in P(\alpha, b, d) : \alpha(u) > b\} \neq \emptyset$. On the other hand, $\forall u \in P(\alpha, b, d)$, we get $b \leq u \leq d$, $-d \leq v \leq 0$, and for $t \in [0, T - l]_{\mathbb{T}}$, from B3,

$$\begin{aligned} \alpha(Fu) &= \int_0^l \phi_q \left(\int_s^T a(r)f(r, u(r), u^\Delta(r))\nabla r \right) \Delta s \\ &\quad + B_0 \left(\phi_q \left(\int_0^T a(r)f(r, u(r), u^\Delta(r))\nabla r \right) \right) \\ &\geq \int_0^l \phi_q \left(\int_0^{T-l} a(r)f(r, u(r), u^\Delta(r))\nabla r \right) \Delta s \\ &\quad + B \left(\phi_q \left(\int_0^{T-l} a(r)f(r, u(r), u^\Delta(r))\nabla r \right) \right) \\ &= (l + B)\phi_q \left(\int_0^{T-l} a(r)f(r, u(r), u^\Delta(r))\nabla r \right) \\ &= (l + B)\frac{b}{lK}\phi_q \left(\int_0^{T-l} a(r)\nabla r \right) = b. \end{aligned}$$

Hence $\alpha(Fu) > b$ for $u \in P(\alpha, b, d)$.

Finally, we show $\alpha(Fu) > b$ for all $u \in P(\alpha, b, d)$ and $\|Fu\| > d$. If $u \in P(\alpha, b, d)$ and $\|Fu\| > d$, then $0 \leq u \leq c$, $-c \leq v \leq 0$, and from B4,

$$\phi_p\left(\frac{M}{m}\right) \int_0^{T-l} a(r)f(r, u(r), u^\Delta(r))\nabla r \geq \int_0^T a(r)f(r, u(r), u^\Delta(r))\nabla r,$$

i.e.

$$\int_0^{T-l} a(r)f(r, u(r), u^\Delta(r))\nabla r \geq \frac{\int_0^T a(r)f(r, u(r), u^\Delta(r))\nabla r}{\phi_p\left(\frac{M}{m}\right)}.$$

Because $Fu \in P$,

$$\begin{aligned}
\alpha(Fu) &= (Fu)(l) \\
&= \int_0^l \phi_q \left(\int_s^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \Delta s \\
&\quad + B_0 \left(\phi_q \left(\int_0^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \right) \\
&\geq l \phi_q \left(\int_0^{T-l} a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \\
&\quad + B \left(\phi_q \left(\int_0^{T-l} a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \right) \\
&= (l+B) \phi_q \left(\int_0^{T-l} a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \\
&\geq (l+B) \phi_q \left(\frac{\int_0^T a(r) f(r, u(r), u^\Delta(r)) \nabla r}{\phi_p \left(\frac{M}{m} \right)} \right) \\
&= \frac{(l+B)m}{M} \phi_q \left(\int_0^T a(r) f(r, u(r), u^\Delta(r)) \nabla r \right) \\
&\geq \frac{(l+B)m}{M} \frac{Fu(0)}{B} \\
&= \left(\frac{l+B}{M} \right) m \frac{\|Fu\|}{BT_0} \\
&\geq \left(\frac{l+B}{M} \right) m \frac{d}{BT_0} \geq b,
\end{aligned}$$

and then $\alpha(Fu) > b$ for all $u \in P(\alpha, b, d)$ and $\|Fu\| > d$. Hence, an application of Theorem 2.3 completes the proof. \square

Theorem 3.2. *Assume that there exist constants a, b, c, d such that $0 < a < b \leq \frac{(l+B)m}{MBT_0}d < d \leq c$ and suppose that f satisfies (B1)–(B3) and*

(C1) $a(t)$ is decreasing for $t \in [0, T]_{\mathbb{T}}$,

(C2) $\phi_p \left(\frac{M}{m} \right) \geq \frac{(T-l)f_M}{lf_m}$,

where

$$f_M = \max\{f(t, u, v)\} \text{ for } (t, u, v) \in [l, T]_{\mathbb{T}} \times [0, c] \times [-c, 0],$$

$$f_m = \min\{f(t, u, v)\} \text{ for } (t, u, v) \in [0, l]_{\mathbb{T}} \times [0, c] \times [-c, 0].$$

Then the boundary value problem (1.1) has at least three positive solutions u_1, u_2 and u_3 satisfying

$$\|u_1\| < a, \quad b < \alpha(u_2), \quad \|u_3\| > a, \quad \alpha(u_3) < b.$$

Proof. We only show $\alpha(Fu) > b$ for all $u \in P(\alpha, b, c)$ and $\|Fu\| > d$. $\forall u \in P(\alpha, b, c)$, we have $0 \leq u \leq c$, $-c \leq v \leq 0$, and from (C1), we get

$$\begin{aligned} \int_0^l a(l)f(r, u(r), u^\Delta(r))\nabla r &\leq \int_0^l a(r)f(r, u(r), u^\Delta(r))\nabla r, \\ \int_l^T a(l)f(r, u(r), u^\Delta(r))\nabla r &\geq \int_l^T a(r)f(r, u(r), u^\Delta(r))\nabla r, \end{aligned}$$

and then

$$\begin{aligned} \frac{\int_l^T a(r)f(r, u(r), u^\Delta(r))\nabla r}{\int_0^l a(r)f(r, u(r), u^\Delta(r))\nabla r} &\leq \frac{\int_l^T a(l)f(r, u(r), u^\Delta(r))\nabla r}{\int_0^l a(l)f(r, u(r), u^\Delta(r))\nabla r} \\ &= \frac{\int_l^T f(r, u(r), u^\Delta(r))\nabla r}{\int_0^l f(r, u(r), u^\Delta(r))\nabla r} \\ &\leq \frac{(T-l)f_M}{lf_m}. \end{aligned}$$

From (C2), we have

$$\phi_p\left(\frac{M}{m}\right) \geq \frac{\int_l^T a(r)f(r, u(r), u^\Delta(r))\nabla r}{\int_0^l a(r)f(r, u(r), u^\Delta(r))\nabla r},$$

and the rest of the proof is similar to final step of Theorem 3.1 so we omit it. \square

4. EXAMPLE

Let $\mathbb{T} = \mathbb{R}$, $T = 1$, $p = 3$, then $T_0 = 1$. Consider the following boundary value problem

$$(4.1) \quad \begin{cases} |u'|u' + a(t)f(t, u(t), u'(t)) = 0, & t \in [0, 1], \\ u(0) - \frac{1}{2}u'(0) = 0, & u'(1) = 0, \end{cases}$$

where

$$\begin{aligned} a(t) &= \begin{cases} 1, & t \in [0, \frac{14}{25}] \\ -t + \frac{4000001}{4000000}, & t \in [\frac{14}{25}, 1], \end{cases} \\ f(t, u, v) &= \begin{cases} 10u^5 + \frac{2+\sin v}{180}, & \text{for } 0 \leq t \leq 1, 0 \leq u \leq 1, -\frac{\pi}{2} \leq v \leq 0; \\ 10u^5 + \frac{1}{180}, & \text{for } 0 \leq t \leq 1, 0 \leq u \leq 1, v \leq -\frac{\pi}{2}; \\ 10\sqrt[20]{u} + \frac{2+\sin v}{180}, & \text{for } 0 \leq t \leq 1, u \geq 1, -\frac{\pi}{2} \leq v \leq 0; \\ 10\sqrt[20]{u} + \frac{1}{180}, & \text{for } 0 \leq t \leq 1, u \geq 1, v \leq -\frac{\pi}{2}. \end{cases} \end{aligned}$$

By the definition of $a(t)$, we know $m = \frac{1}{2000}$ and $M = 1$. It is obvious that $A = B = \frac{1}{2}$. Choose $l = \frac{11}{25}$, a direct calculation shows that

$$K = \left(\frac{l+B}{l}\right)\phi_q\left(\int_0^{T-l} a(r)dr\right) = \frac{(\frac{11}{25} + \frac{1}{2})}{\frac{11}{25}}\sqrt{\frac{14}{25}} = \frac{47}{22}\sqrt{\frac{14}{25}}.$$

If we take $a = \frac{1}{2}$, $b = 1$, $d = 1250$, $c = 1260$, then

$$0 < \frac{1}{2} < 1 < \frac{\left(\frac{11}{25} + \frac{1}{2}\right) \times \frac{1}{2000}}{\frac{1}{2}} \times 1250 < 1250 < 1260,$$

we have

$$\phi_3\left(\frac{a}{M}\right) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}, \quad \phi_3\left(\frac{c}{M}\right) = 1260^2, \quad \phi_3\left(\frac{b}{lK}\right) = \frac{25 \times 22^2 \times 25^2}{11^2 \times 47^2 \times 14},$$

then the nonlinear term f satisfies

$$(B1) \quad f(t, u, v) \leq 10 \times \left(\frac{1}{2}\right)^5 + \frac{1}{90} < \frac{1}{4} = \phi_3\left(\frac{a}{M}\right),$$

$$\text{for } 0 \leq t \leq 1, \quad 0 \leq u \leq \frac{1}{2}, \quad -\frac{1}{2} \leq v \leq 0;$$

$$(B2) \quad f(t, u, v) \leq 10 + \frac{1}{60} < 1260^2 = \phi_3\left(\frac{c}{M}\right),$$

$$\text{for } 0 \leq t \leq 1, \quad 0 \leq u \leq 1, \quad -1260 \leq v \leq 0;$$

$$f(t, u, v) \leq 10 \sqrt[20]{1260} + \frac{1}{60} < 1260^2 = \phi_3\left(\frac{c}{M}\right),$$

$$\text{for } 0 \leq t \leq 1, \quad 1 \leq u \leq 1260, \quad -1260 \leq v \leq 0;$$

$$(B3) \quad f(t, u, v) \geq 10 + \frac{1}{180} > \frac{25 \times 22^2 \times 25^2}{11^2 \times 47^2 \times 14} = \frac{31250}{19963} = \phi_3\left(\frac{b}{lK}\right)$$

$$\text{for } 0 \leq t \leq \frac{14}{25}, \quad 1 \leq u \leq 1250, \quad -1250 \leq v \leq 0;$$

$$(B4) \quad \min\{f(t, u, v)\} = \frac{1}{180} \text{ for } 0 \leq t \leq 1, \quad 0 \leq u \leq 1260, \quad -1260 \leq v \leq 0;$$

$$\max\{f(t, u, v)\} = 10 \sqrt[20]{1260} + \frac{1}{60} \text{ for } 0 \leq t \leq 1, \quad 0 \leq u \leq 1260, \quad -1260 \leq v \leq 0.$$

$$\phi_3\left(\frac{M}{m}\right) = 2000^2, \quad \int_0^{1-l} a(t) dt = \frac{14}{25},$$

$$\int_0^{\frac{14}{25}} 1 dt + \int_{\frac{14}{25}}^1 \left(-t + \frac{4000001}{4000000}\right) dt = \frac{14}{25} + \frac{9680011}{100000000} = \frac{65680011}{100000000}.$$

We have

$$\min\{f(t, u, v)\} \phi_3\left(\frac{M}{m}\right) \int_0^{1-l} a(t) dt \geq \max\{f(t, u, v)\} \int_0^1 a(t) dt$$

$$\text{for } (t, u, v) \in [0, 1] \times [0, 1260] \times [-1260, 0].$$

$$\frac{1}{180} \times 2000^2 \times \frac{14}{25} > \left(10 \sqrt[20]{1260} + \frac{1}{60}\right) \times \left(\frac{65680011}{100000000}\right).$$

Thus by Theorem 3.1, we find that boundary value problem (4.1) has at least three positive solutions.

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