



Evaluation of Soft Computing Methods for Estimating Tangential Young Modulus of Intact Rock Based on Statistical Performance Indices

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Abstract The tangential Young modulus (E_{ti}) of intact rock is a critical parameter in engineering geological design calculations and rock mass classification systems. The E_{ti} of various rock types has been successfully estimated by many studies based on numerous soft computing methods in recent years. However, these studies mainly involve a single analysis method or are valid for a limited number of samples. For this reason, this study aimed to compare artificial neural networks (ANN), adaptive neural fuzzy inference system (ANFIS), and Gene expression programming (GEP) methods to estimate the E_{ti} of various rock types based on 147 datasets collected from the published literature. As a result of the soft computing analyses, three different predictive models were proposed in this study. In the proposed prediction models, rock properties such as dry density (ρ_d), effective porosity (n_e), P-wave velocity (V_p), and uniaxial compressive strength (UCS) were used. The estimation performance of the proposed models was examined through several performance indices such as coefficient of determination (R^2), root mean square error (RMSE), the variance accounted for

(VAF), and mean absolute percent error (MAPE). As a result of statistical analyses, it was determined that the ANFIS model presents a better prediction performance ($R^2=0.967$) than the other methods in the training datasets. On the other hand, the accuracy of the ANFIS model decreased significantly in the test datasets ($R^2=0.803$). Furthermore, the GEP model presented the lowest predictive performance. Finally, considering the overall estimation accuracy of the proposed models, it was concluded that the proposed ANN model with an R^2 of 0.94 could reliably be used to estimate the E_{ti} of investigated rocks.

Keywords Tangential Young modulus · Intact rock · Soft computing · Performance indices

1 Introduction

Tangential Young modulus of intact rock (E_{ti}) is a critical parameter widely used in estimating the rock mass elastic modulus as an input parameter (Nicholson and Bieniawski 1990; Mitri et al. 1994; Kayabasi et al. 2003; Gökceoglu et al. 2004; Ramamurthy 2004; Sönmez et al. 2004; Sönmez et al. 2006; Hoek and Diederichs 2006;). It is also used to perform numerical modeling of rock masses (Bidgoli et al. 2013; Xu et al. 2016; Shu et al. 2019). Based on different physical and mechanical rock properties, several researchers employed single and multiple regression analyses to estimate E_{ti} of various rock

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types (Dinçer et al. 2004; Moradian and Behnia 2009; Yılmaz and Yüksek 2009; Armaghani et al. 2016).

Although regression-based predictive models are an easy and practical way to estimate the E_{ti} , they frequently fail to capture some gaps and uncertainties in the dataset (Dehghan et al. 2010; Behzadafshar et al. 2019; Roy and Singh 2020). Furthermore, the inherent drawbacks in rock heterogeneity are directly related to the host rock's origin, geodynamic, and tectonic history. Therefore, most regression-based predictive models are site-specific and valid for a specific area of interest (Pappalardo 2015).

On the other hand, soft computing tools such as fuzzy inference system (FIS), artificial neural networks (ANN), gene expression programming (GEP), and support vector machine (SVM) are relatively more successful tools in dealing with the uncertainties and outliers in the dataset (Kecman 2001; Campbell and Ying 2011). For instance, a FIS model was established by Gökçeoğlu and Zorlu (2004) to estimate the E_{ti} of several rocks from Turkey. Additionally, from the rock mechanics and rock engineering perspective, ANN has gained popularity to estimate the E_{ti} of different rock materials and rock masses (Sönmez et al. 2006; Tiryaki 2008; Yılmaz and Yüksek 2009; Kahraman et al. 2009; Dehghan et al. 2010; Torabi-Kaveh et al. 2015; Köken 2021). To take advantage of both FIS and ANN techniques, a hybrid model, called adaptive neuro-fuzzy inference system (ANFIS) was also used to evaluate the deformation properties of rocks (Yılmaz and Yüksek 2009; Singh et al. 2012, 2017; Armaghani et al. 2015; Roy and Singh 2020).

Based on the above-mentioned studies focusing on several soft computing methodologies, the common argument is that the performance of the soft computing methods is better than regression-based models in estimating the E_{ti} . Besides, regression-based predictive models are sensitive to the inter-item correlation.

Therefore, the strong interaction between the predictors can lead to bias on the output and exaggerate the strength of the predictive model (Nicodemus and Malley 2009). Therefore regression-based predictive models were not addressed in this study.

In the light of the above explanations, the present study introduces several predictive models to estimate the E_{ti} of rocks based on three widely used soft computing methods (ANN, ANFIS, and GEP). For this purpose, a comprehensive literature survey was

conducted to compile such datasets to implement these methods. In this regard, dry density (ρ_d), effective porosity (n_e), P-wave velocity (V_p), and uniaxial compressive strength (UCS) values of different rock types were used as input parameters. Since the above-mentioned intact rock parameters are routinely measured in many rock engineering projects, it is cost-effective to estimate E_{ti} of different rock types from these parameters. However, it should be noted that the most appropriate model to estimate E_{ti} of rocks may change depending on the strength of the relationship between the predictors and predicted parameters. For this reason, different techniques have to be compared when assessing the E_{ti} of rocks.

As a result, three different soft computing-based predictive models were established to estimate the E_{ti} of considered rocks in this study. Based on several statistical indicators, the performance of the proposed predictive models was compared to one another to choose the most accurate model for evaluating the E_{ti} . The details and mathematical formulations behind the proposed models were also given in this study to let users implement them more efficiently. The present study in this regard can save time and provide accurate and practical information on the E_{ti} of different rock types.

2 Materials and methods

2.1 Variable selection

Deformability, like the strength and abrasion properties of rocks, depends mainly on the mineralogical composition, porosity, and the degree of fracturing of the rock material. In this regard, the V_p is closely related to the elastic properties of rock materials (Goodman 1989) and well correlated with the E_{ti} of rocks (Yaşar and Erdoğan 2004; Moradian and Behnia 2009; Yılmaz Yüksek 2009; Pappalardo 2015). In addition, sensitivity analyses performed by Abdulhadi and Barghouthi (2012) demonstrated that the V_p is the most influential parameter on the E_{ti} of rocks.

In general terms, pores and joints are the weakest and most deformable elements in the rock (Abdulhadi and Barghouthi 2012). Therefore, the n_e can also be declared a correlative parameter to estimate the E_{ti} of rocks (Lashkaripour 2002; Heidari et al. 2010; Yağız

2011). Similarly, ρ_d is another effective parameter on the E_{ti} . However, sometimes strong correlations can not be obtained between the ρ_d and E_{ti} (Zhang 2016). On the other hand, the UCS has not mainly been used as an independent variable for the evaluation of E_{ti} . The underlying reason for this phenomenon comes from the fact that the sample preparation for the UCS test is laboring. It is valid for very weak, fragile, and bim rocks; however, this test is routinely carried out in almost every engineering geological project. Therefore, several researchers documented strong relationships between the UCS and E_{ti} of rocks (Lashkaripour 2002; Shalabi et al. 2007; Pappalardo 2015). In the light of the above explanations, rock properties such as the ρ_d , n_e , V_p , and UCS were selected as input parameters in ANFIS, GEP, and ANN analyses. Nevertheless, Pearson’s correlation (r) analyses also revealed the single relationships or correlations between those input parameters and E_{ti} in this study.

2.2 Database development

A comprehensive literature survey was conducted to establish several predictive models based on ANFIS, GEP, and ANN methods. As a result of the literature survey, a database composed of 147 datasets, including the considered rock properties (ρ_d , n_e , V_p , UCS, and E_{ti}), was collected (Table 1). Unfortunately, a significant number of previous studies could not be regarded due to the lack of quantitative information on input parameters considered in this study. The considered dataset was randomly divided into parts of training

(70/100) and testing (30/100) in all soft computing methods.

2.3 Statistical indicators

To evaluate the performance of the predictive models, several statistical indicators such as the coefficient of determination (R^2), root mean square error (RMSE), mean absolute percentage error (MAPE), and the variance accounted for (VAF) were adopted in this study. These error metrics target different requirements depending on data sets, and hence they can be used together to evaluate the errors from different aspects. The mathematical expressions of these indicators are given by Eqs. 1–4.

$$R^2 = \left(\frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}} \right)^2 \tag{1}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (y_i - x_i)^2}{n}} \tag{2}$$

$$MAPE = \frac{1}{n} \sum_{i=1}^n |y_i - x_i| \tag{3}$$

$$VAF = \left(1 - \frac{\text{var}(y_i - x_i)}{\text{var}(y_i)} \right) \times 100 \tag{4}$$

Table 1 Datasets considered in this study

Rock type	ρ_d (g/cm ³)	n_e (%)	V_p (km/s)	UCS (MPa)	E_{ti} (GPa)	n	Reference
Travertine, Limestone, Schist	2.31–2.78	0.16–2.05	3.80–5.60	32.00–114.00	22.00–57.00	9	Yagiz (2011)
Peridotite	3.12–3.40	0.06–0.26	7.05–7.98	65.21–241.56	26.40–69.30	35	Diamantis et al. (2011)
Basaltic rocks	1.72–2.71	0.91–12.95	2.61–5.93	17.20–145.15	5.20–66.24	18	Karakuş and Akatay (2013)
Andesite	2.29–2.65	4.37–16.30	3.63–4.55	60.13–211.05	19.90–43.70	22	Siratovich et al. (2014)
Limestone, Dolomite	2.57–2.80	2.40–10.36	3.30–6.32	15.18–112.00	2.37–18.80	25	Pappalardo (2015)
Basaltic rocks	2.33–3.00	0.79–6.64	2.72–6.55	24.78–139.12	7.14–40.48	8	Singh et al. (2017)
Sandstone	2.12–2.50	5.77–14.51	1.35–2.86	19.94–116.68	2.93–14.83	6	Abdi et al. (2018)
Sandstone	2.41–2.72	0.79–7.86	2.52–4.62	54.19–166.57	10.52–37.20	24	Köken (2020)

ρ_d : Dry density, n_e : Effective porosity, V_p : P-wave velocity, UCS Uniaxial compressive strength, E_{ti} : Tangential Young modulus, n Number of datasets

where x is the dependent variable, y is the independent variable, and n is the number of datasets.

3 Data analysis methods

3.1 Pearson’s correlation analyses

In this section, the single correlations between the input parameters and E_{ti} were investigated through Pearson’s correlation analysis. The Pearson’s correlation coefficient (r) is calculated using Eq. 5.

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n \sum x^2 - (\sum x)^2][n \sum y^2 - (\sum y)^2]}} \quad (5)$$

where x is the dependent variable, y is the independent variable.

Based on the above explanations, Pearson’s correlation matrix for the considered variables is listed in

Table 2 Pearson’s correlation matrix for the evaluation of E_{ti} in this study

Variable	ρ_d	n_e	V_p	UCS
E_{ti}	0.647	− 0.594	0.663	0.782

* p values were found to be lower than 0.0005

Fig. 1 Typical ANFIS architecture (Singh et al. 2012)

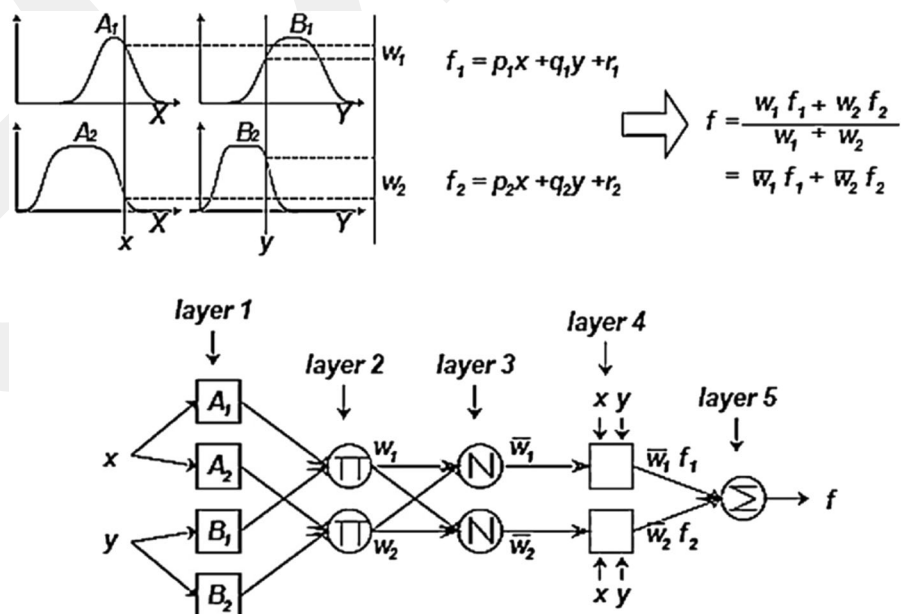


Table 2. Accordingly, one can claim that the considered rock properties can be highly correlated with the E_{ti} . Therefore, all these parameters were integrated into the artificial intelligence methods considered in this study.

3.2 Adaptive fuzzy inference system (ANFIS)

Considering many advantages, researchers have used ANFIS to build such predictive models that are used in many engineering geological problems (Roy and Singh 2020; Yeşiloğlu-Gültekin et al. 2013; Sharma et al. 2017). The advantage of the ANFIS is that it practices a hybrid learning process to estimate the premise and consequent parameters (Jang 1992). In this context, Sugeno fuzzy reasoning algorithm based on numerous membership functions is primarily adopted in most ANFIS models (Fig. 1). From this approach, several ANFIS models were created in the MATLAB environment in this study, and one feasible ANFIS model was proposed in the context of the ANFIS model development title.

3.3 Gene expression programming (GEP)

The GEP is an evolutionary-based algorithm that produces an explicit mathematical formulation series between dependent and independent variables. The GEP was first developed by Ferreira (2001), and for

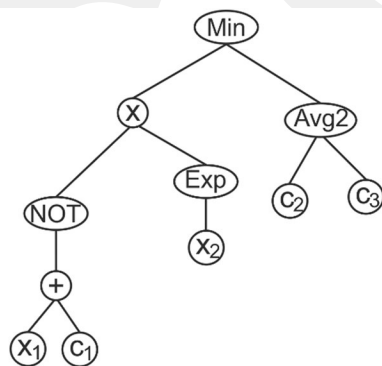
the past two decades, the GEP has gained popularity among researchers in various engineering fields. The GEP algorithm functions using some independent variables (x_1, x_2), constants (c_1, c_2) and some numerical operators (+, −, x/, min, max, exp etc.). It tries to find some empirical formulae based on some sub-expression trees (Sub-ETs), an example of which is given in Fig. 2. Based on the SubETs like Fig. 2, the GEP generates several SubETs and combines them with a linking function (e.g., addition or multiplication). Finally, it creates a predictive model for the relevant dependent variable.

The GEP analyses were performed using GeneXproTools (v.5.0.3902) software in this study. Several number of chromosomes, head sizes, and numerical operators were attempted to obtain the most feasible predictive model.

3.4 Artificial neural networks (ANN)

ANN can analyse the data, learn and save the experience-based knowledge, and utilize it in future predictions (Singh et al. 2001; Rabbani et al. 2012). This parallel distributed learning algorithm is applicable to many problems, from social sciences to applied sciences. In most ANN models, feed-forward backpropagation algorithm is adopted (Fig. 3).

Although ANN can map the input–output relationships, they are mainly black-box models, limiting their broader usage. Several ANN analyses were



Mathematical formula of the above sub-expression tree

$$\text{Min} ((1 - (x_1 + c_1)) \times \exp(x_2); (c_2 + c_3)/2)$$

Fig. 2 Illustration of a sub-expression tree in a GEP analysis

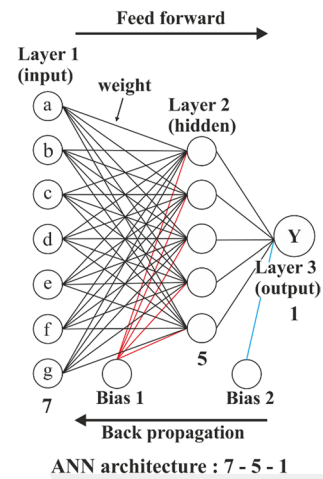


Fig. 3 Example of the feed-forward backpropagation algorithm in an ANN model

performed in this study using nntool in the MATLAB environment. The novel ANN-based predictive model was introduced with definite mathematical equations using the weights and biases extracted from the ANN analyses.

4 Results and discussion

4.1 Proposed ANFIS model

In the developed ANFIS model, seven Gaussian membership functions represented each input parameter (ρ_d, n_e, V_p and UCS). Depending upon these membership functions, seven if–then rules were created in the context of the ANFIS analyses. The ANFIS analyses continued until the minimum RMSE values were achieved. Some illustrations of the proposed ANFIS model in the MATLAB environment are given in Fig. 4. The predicted and measured E_{ti} values for the proposed ANFIS model are also given in Fig. 5. Accordingly, the coefficient of determination (R^2) value for the proposed ANFIS model was found to be 0.93.

4.2 Proposed GEP model

In this section, novel applications of GEP were introduced to establish a predictive model for the evaluation of E_{ti} . For this purpose, the GeneXpro software

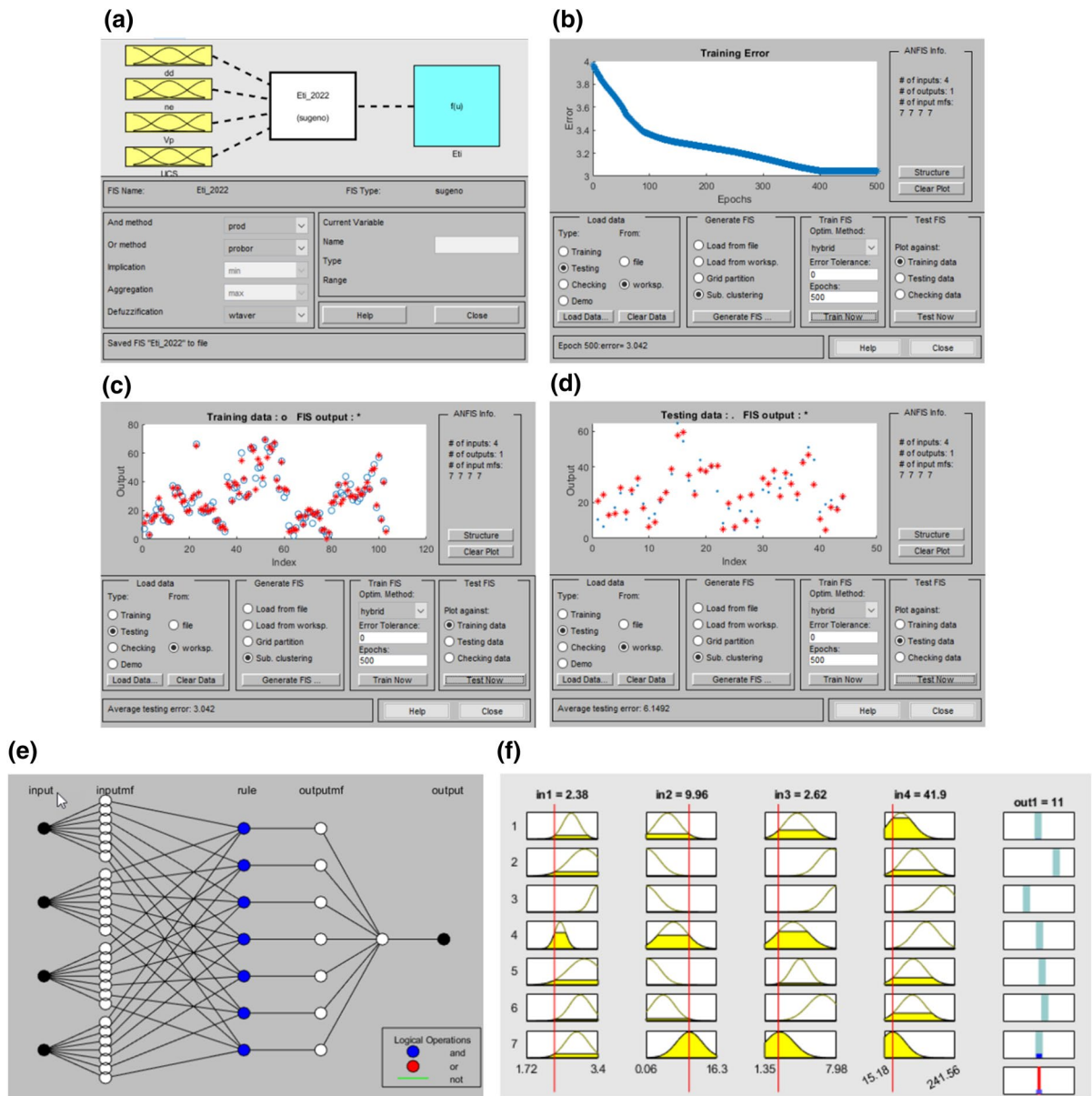


Fig. 4 Illustrations of the proposed ANFIS model in the MATLAB environment **a** Input parameters **b** Training process **c** Scatter plot of training data **d** Scatter plot of test data **e** ANFIS architecture **f** ANFIS rule viewer

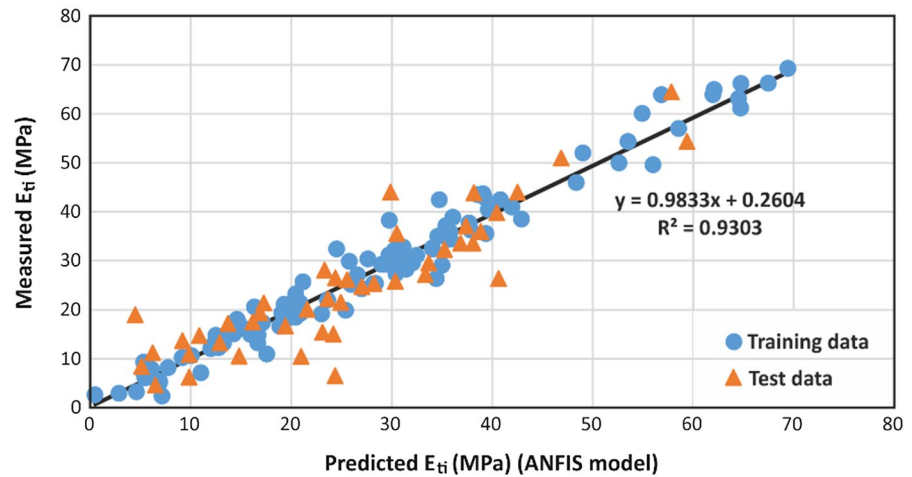
was used to implement various GEP models. In these models, the number of chromosomes, head sizes, and gene sizes were assigned to 20, 8, and 3, respectively. The linking function was the multiplication, and RMSE (Eq. 2) was regarded as the fitness function. Before performing GEP analyses, the dataset (Table 1) was normalized between 0 and 1, using Eq. 6.

$$V_n = \frac{x_i - x_{\min}}{x_{\max} - x_{\min}} \tag{6}$$

where x_i is the relevant parameter to be normalized, x_{\min} , and x_{\max} are the minimum and maximum values in the dataset (Table 1).

As a result of the GEP analyses, the sub-expression trees obtained from the GEP analyses are given

Fig. 5 Predicted and measured E_{ti} values for the proposed ANFIS model



in Fig. 6. Based on these sub-expression trees, the E_{ti} can be estimated by Eq. 7. Furthermore, the mathematical expressions of the subexpression trees (Sub-ETs) and normalization functions of the GEP model are listed in Table 3. Finally, the predicted and measured E_{ti} values for the proposed GEP model are also given in Fig. 7. Accordingly, the R^2 of the proposed GEP model is about 0.84.

$$E_{ti} = 66.92 \prod_{i=1}^3 x_i + 2.38 \tag{7}$$

4.3 Proposed ANN model

The artificial neural network (ANN) has been widely adopted to predict several dependent variables based on complex datasets. It is a well-accepted method in most engineering geological problems. In this study, the neural network toolbox (nntool) was utilized to establish several neural networks in the MATLAB environment. Various possible network architectures with variable hidden layers and neurons were attempted to determine the most reliable ANN structure.

For estimating the E_{ti} , the most convenient ANN architecture was found to be 4–6–1 (Fig. 8). To increase training efficiency during ANN analyses, just like the GEP model, the dataset was also normalized between –1 and 1, using Eq. 8.

$$V_n = 2 \left(\frac{x_i - x_{\min}}{x_{\max} - x_{\min}} \right) - 1 \tag{8}$$

where x_i is the relevant parameter to be normalized, x_{\min} , and x_{\max} are the minimum and maximum values in the dataset (Table 1).

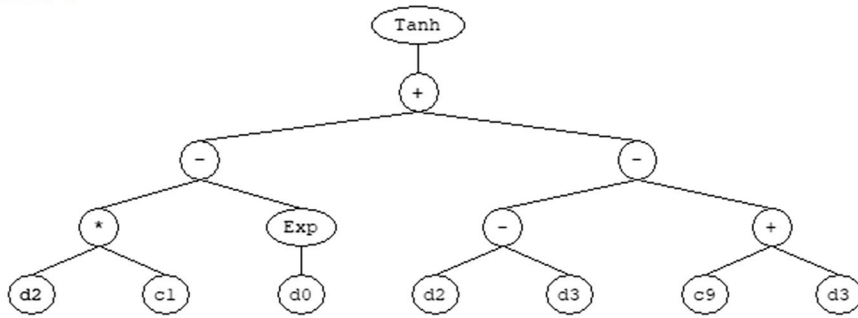
As a result of the ANN analyses, the E_{ti} can be estimated using Eq. 9. The subfunctions of Eq. 9 were determined based on the deterministic approach described by Das (2013), and they are listed in Table 4. According to the ANN analyses results, the predicted and measured E_{ti} values are given in Fig. 9. One can claim from Fig. 9 that the predicted and measured E_{ti} values are in good agreement. The R^2 of this model is about 0.94, which can yield the most promising results among the soft computing methods adopted in this study.

$$E_{ti} = 32.439 \tanh \left(\sum_{i=1}^6 x_i - 0.60235 \right) + 35.646 \tag{9}$$

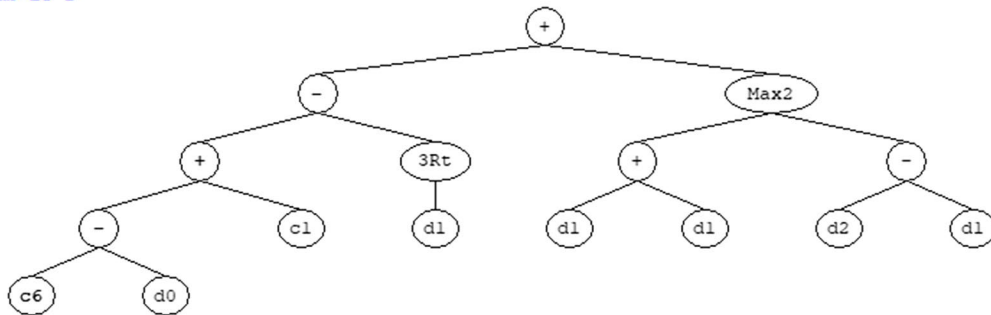
4.4 Performance evaluation

The performance of the proposed models was also evaluated using several statistical indices, which were previously explained in Sect. 2.3. Accordingly, the performance evaluations of the models are listed in Table 5.

Sub-ET 1



Sub-ET 2



Sub-ET 3

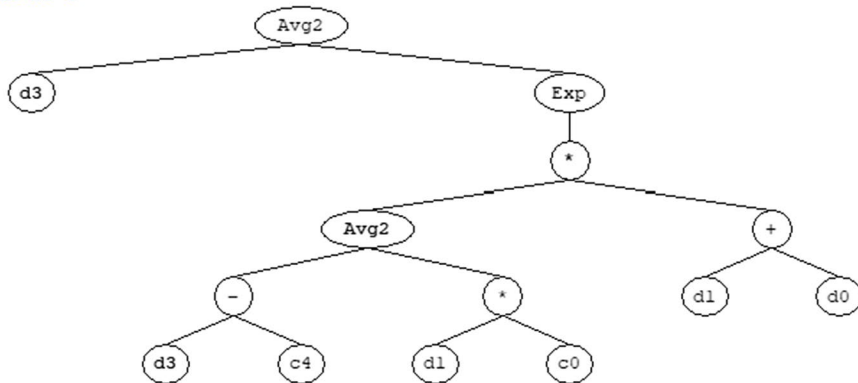


Fig. 6 Sub expression trees of the proposed GEP model (d0: ρ_d , d1: n_e , d2: V_p , d3: UCS, g1c1: 4.8890, g1c9: -1.5321, g2c1: 4.6835, g2c6: -2.8698, g3c4: 3.8826, g3c0: -4.2235).

Theoretically, a predictive model having RMSE and MAPE values of 0 and VAF of 100 is assumed to have the perfect prediction performance. In this regard, the performance of the proposed models in this study is higher than most of the reviewed literature (Gökçeoğlu and Zorlu 2004; Yağız et al. 2012;

Beiki et al. 2013; Armaghani et al. 2015; Singh et al. 2017).

Similar prediction performances to estimate the E_i of various rock types have also been reported by Tiryaki (2008) and Behzadafshar et al. (2019), who used different artificial intelligence methods such as

Table 3 Mathematical expressions of the subexpression trees and normalization functions of the proposed GEP model

$$x_1 = \tanh(5.8890^n V_p - \exp({}^n \rho_d) + 1.5321 - 2^n UCS)$$

$$x_2 = 1.8138 + {}^n \rho_d - ({}^n n_e)^{1/3} + \max(2^n n_e; ({}^n V_p - {}^n n_e))$$

$$x_3 = \frac{{}^n UCS + \exp\left(\frac{({}^n UCS - 3.8827) - 4.2235^n n_e}{2}\right) ({}^n \rho_d + {}^n n_e)}{2}$$

Normalization functions

$${}^n \rho_d = 0.5952 \rho_d - 1.0238 {}^n n_e = 0.0616 n_e - 0.0037$$

$${}^n V_p = 0.1508 V_p - 0.2036 {}^n UCS = 0.0044 UCS - 0.0671$$

regression tree (RT) and ANN trained with genetic algorithm (GA-ANN).

In contrast, the ANFIS model proposed by Roy and Singh (2020) and the ANN models proposed by Madhubabu et al. (2016) and Torabi-Kaveh et al. (2015), ANN optimized with the cuckoo optimization algorithm (COA-ANN) model proposed by Mokhtari and Behnia (2019) have better prediction performances than the models proposed in this study. However, it should be noted that except for Madhubabu et al. (2016), the others have studied granite, limestone in one locality, and coal measures rocks of close localities, respectively. In addition, Madhubabu et al. (2016) studied a variety of carbonate rocks, but they used Poisson’s ratio (ν) as a predictor in their model, which significantly increases the model accuracy. However, if the ν of rocks can be measured in the laboratory, in the meantime, the E_{ti} can also be measured using the same apparatus measuring the deformations in/on rock such as strain gauges, strain jackets, or linear variable differential transformers

Fig. 7 Predicted and measured E_{ti} values for the proposed GEP model

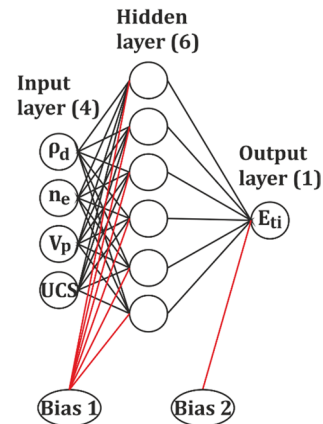
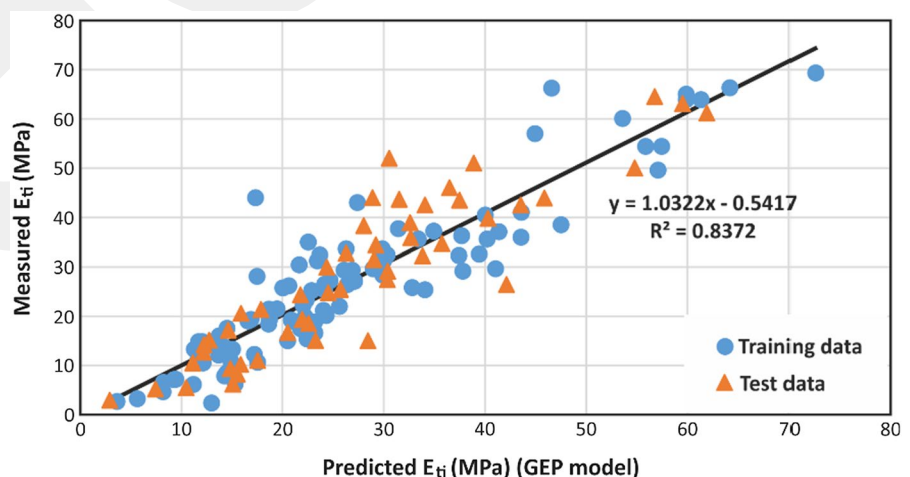


Fig. 8 ANN architecture adopted in this study

(LVDTs). Therefore, the fundamental need for predictive modeling loses its significance when adopting some parameters, which can be obtained in the same or relevant testing methods.

The proposed GEP model (Fig. 7) showed the lowest prediction performance among the investigated methods in this study. Nevertheless, Beiki et al. (2013) obtained similar performance for their dataset using the GEP methodology. However, they did not compare the performance of GEP with other artificial intelligence methods.

Several researchers also reported that ANFIS performs better prediction performances than the other soft computing approaches do (Yılmaz and Yüksek 2009; Yeşiloğlu-Gültekin et al. 2013; Sharma et al. 2017; Roy and Singh 2020). However, in this study,

Table 4 Subfunctions of the proposed ANN model

$$\begin{aligned}
 x_1 &= 0.80591 \tanh (1.3352^n \rho_d - 2.8848^n n_e + 0.5939^n V_p - 1.2781^n UCS - 3.4412) \\
 x_2 &= 1.2536 \tanh (-2.4135^n \rho_d - 0.54488^n n_e + 2.4974^n V_p + 1.9566^n UCS - 1.6231) \\
 x_3 &= 1.9881 \tanh (-2.2059^n \rho_d + 0.62106^n n_e + 0.23545^n V_p + 1.3404^n UCS + 0.24292) \\
 x_4 &= -0.64949 \tanh (0.36941^n \rho_d + 3.9073^n n_e + 4.6696^n V_p - 1.3648^n UCS + 1.3133) \\
 x_5 &= 0.39642 \tanh (5.7459^n \rho_d - 2.7388^n n_e + 1.4168^n V_p + 5.2252^n UCS + 3.8834) \\
 x_6 &= -3.107 \tanh (-1.9314^n \rho_d - 0.19387^n n_e - 0.85966^n V_p + 1.5011^n UCS - 0.86693)
 \end{aligned}$$

Normalization functions

$$\begin{aligned}
 {}^n \rho_d &= 1.1905 \rho_d - 3.0476 {}^n n_e = 0.1232 n_e - 1.0074 \\
 {}^n V_p &= 0.3017 V_p - 1.4072 {}^n UCS = 0.0088 UCS - 1.1341
 \end{aligned}$$

Fig. 9 Predicted and measured E_{ti} values for the proposed ANN model

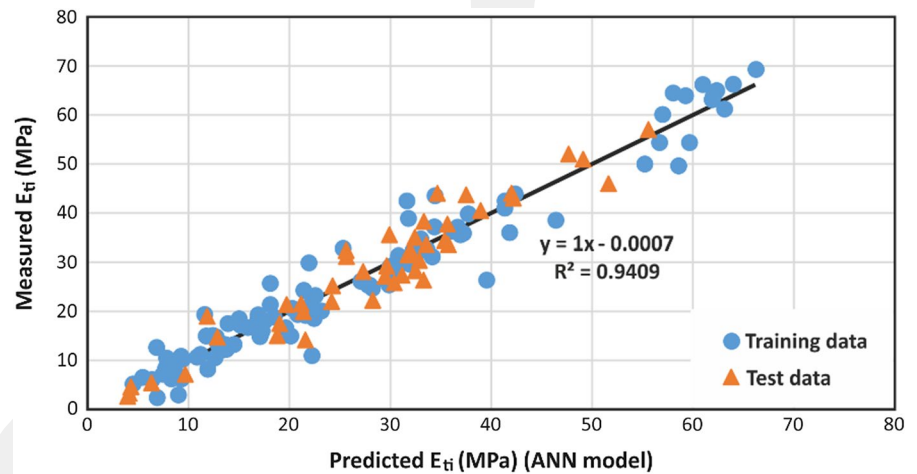


Table 5 Performance evaluation of the proposed models

Performance index		Data analysis method		
		ANFIS	GEP	ANN
R^2	Training	0.967	0.848	0.947
	Testing	0.803	0.820	0.917
	Overall	0.930	0.837	0.940
RMSE	Training	3.042	6.152	3.906
	Testing	6.149	7.023	3.795
	Overall	4.219	6.455	3.873
MAPE	Training	2.315	4.531	2.860
	Testing	4.691	5.301	2.957
	Overall	3.026	4.788	2.889
VAF	Training	96.70	84.80	94.70
	Testing	79.67	81.47	91.66
	Overall	93.00	83.63	94.09

ANFIS was found to have better learning capability while not reaching the intended accuracy in testing

data. More profoundly, the ANFIS model has a better generalization ability than the GEP model, but then it presents a lower performance than the ANN model. Consequently, considering the overall accuracy, the ANN model provided the best performance with an R^2 of 0.94 in predicting the E_{ti} of rocks (Table 5).

5 Conclusions

The E_{ti} of rocks is a fundamental parameter to figure out deformation behavior and perform numerical modeling of rock masses. Although soft computing methods have been increasingly used to predict E_{ti} of various rock types, there is still a need for predictive models, which have better generalization ability to represent a wide range of rock types. From this point of view, the present study aimed to establish some predictive models for evaluating the E_{ti} of different rocks based on ANFIS, ANN, and GEP methods and

compare them with several statistical indicators. For this purpose, a comprehensive literature survey was conducted (Table 1) to compile a large number of datasets. Based on the rock properties of ρ_d , n_c , UCS and V_p , three different predictive models were proposed to estimate the E_{ti} of rocks in this study.

Based on the prediction performance indices such as R^2 , RMSE, MAPE and VAF, it was determined that the ANFIS presented the highest learning ability, whereas it failed when considering the testing of the dataset. On the other hand, the GEP model (Eq. 7) presented the lowest performance in estimating the E_{ti} of rocks. The ANN method (Eq. 9) had the highest overall accuracy in predicting the E_{ti} of rocks (Table 5).

The present study in this regard can be declared as a comprehensive study showing the applicability of ANFIS, GEP, and ANN methods for the evaluation of E_{ti} for different rock types. The details and mathematical formulations behind the established predictive models were also introduced in this study to let users implement them more efficiently. As a consequence, the ANN-based predictive model (Eq. 9) was found to have the optimal approximation to the E_{ti} of different rock types. However, the number of datasets and data analysis methods should be increased to observe the similarities or difficulties of modeling the E_{ti} of different rock types as a function of different rock properties.

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Data availability Enquiries about data availability should be directed to the authors.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

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