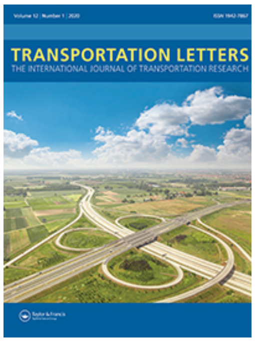

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Effects of total cost of ownership on automobile purchasing decisions

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ABSTRACT

In this paper, we reveal a complete picture of ownership-related expenses and construct a decision model which helps decision maker to make the best choice when purchasing an automobile. The decision model helps the customers to understand what a car will cost beyond its purchase price when customers consider out-of-pocket expenses like fuel, repair, and insurance. Moreover, decision maker's preferences need to be elicited thowever, elicitation of these preferences is difficult when preferential dependencies exist or possible number of uncertainties is high. Therefore, we approximate representative joint probability distributions of a decision maker with partial information. We use a database of sedan models of all automobile brands and run a simulation to analyze the total cost of ownership of driving a car for 5 years. We found that even more expensive car could save more money over the first 5 years of ownership.

KEYWORDS

Partial information; entropy; cumulative residual entropy; maximum entropy; maximum cumulative entropy

Introduction

Car is one of the most important and costly investment people may make, and so it is important to get the best one that is right for the customers. While decision maker is deciding which car to buy, s/he needs to consider so many interior and exterior features of a car at the same time. Most of the interior and exterior features of the cars in the same class are almost identical. Fuel economy varies by only a few miles per gallon (MPG) in most cases given the same classes of engines, and most of these cars come with similar features. Size and space are also close with only a few inches difference between them in most cases, and the gap has narrowed tremendously in quality in the past few years.

Car buyers focus solely on the manufacturer's recommended sale price (MRSP) because of similar interior and exterior features for brand new cars. Car buyers just assume that the selling price is all they will need to know when finding the best deals and cost of the car. It is comparatively easy to see which car cost more to buy initially by comparing the MRSP. But it is harder to know what that car really costs to own because there are several further expenses critical to determine the overall cost of owning a car over time like the repair cost, depreciation cost, and insurance cost. Therefore, it is important to take into account these kinds of uncertainties to make a better-informed decision when deciding what car the decision maker can or cannot afford in the long run.

In real life decisions, however, the amount of information we can collect from the decision maker is limited because eliciting more information from the decision maker is difficult, time consuming, expensive, and also decision maker could be unable or unwilling to make decisions. Therefore, decisions need to be made with partial information. There are several studies that try to help decision maker to come up with a decision with partial information. In most of these studies, the probability distributions are approximated by some simplifying assumptions such as probability independence (Keeney 1973; Fishburn and Keeney 1974; Bordley and Multiplicative 1982). Assuming probability independence simplifies the process; however, the accuracy of the approximate probability distributions reduces due to the loss of information. More accurate probability distributions can be approximated if probability dependence is incorporated among variables. Several methods have been proposed to incorporate dependence between the variables in the literature (Chow and Liu 1968; Clemen and Reilly 1999; Clemen, Fischer, and Winkler 2000; Abbas 2006; Kelly 2007). In this study, we approximate joint probability distributions by incorporating probability dependence into the decision model and propose a new approximation approach to incorporate dependence between attributes.

The remainder of this paper is structured as follows: The 'motivation and background' section presents motivation and background of the study. The 'model and method: true cost to own an automobile' section defines the model and method of total cost of ownership of an automobile. The 'approximation methods of joint probability distributions' section discusses the maximum cumulative residual entropy (CRE) and CRE-based dependence tree. The 'results of the analysis' section presents the results of Monte Carlo simulation to quantify and compare the accuracy of approximation methods.

Motivation and background

In this study, we bring out a complete picture of ownership-related expenses that is designed to help consumers make the right choice when purchasing an automobile. While comparing two cars, some of the features are easy to compare. For instance, assume a customer has narrowed the list to two cars: Car A and Car B. If Car A gets 30 mpg and Car B gets 25 mpg, the customer will have to fill up more often and spend more money at the pump with Car B. But it is not as easy as to compare several cost-related features such as depreciation. If the depreciation for Car B is better than Car A, then Car B has a higher resale value after 5 years. So, our aim in this study is to help the customers to understand what a car will cost beyond its purchase price when you consider out-of-pocket expenses like fuel, insurance, reliability (maintenance and repair), mpg, and the car's resale value. We recommend an optimum car to the decision maker which fits the budget and

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preferences specified by the car buyer. Each car is recommended to the decision maker based on how well it matches the decision maker's preferences.

We also integrate approximation of joint distributions into the total cost of ownership of an automobile case. We propose methods to approximate joint probability distributions without probability independence assumption. The main contribution of our approach is to approximate joint probability distribution of a set of discrete random variables using its lower-order assessments based on CRE (Sutcu 2015), which is an alternative measure of entropy using cumulative probability distributions (Rao et al. 2004).

In our search of literature, despite the importance of the car ownership costs, relatively little research has been conducted on the impacts of the total cost of car ownership. Greenlees (1980) measured the impact of the price of gasoline on the mix of new automobile purchases. Kayser (2002) analyzed the relationship between gasoline demand and car choice. Barnes and Langworthy (2004) develop some simple parameters to help decision maker understand how vehicle operating costs change as a result of the new conditions created on highways. Jong et al. (2004) examined the effects of both fixed and variable car cost changes on both car ownership and use in the Netherland. Van Vliet et al. (2010) calculated efficiency, fuel consumption, and total costs of ownership and greenhouse gas emissions for economic comparison of series hybrid, plug-in hybrid, fuel cell, and regular cars. Gilmore and Lave (2013) employed data from used vehicle auctions in 2008–2009 to compare the difference in resale prices to the expected fuel savings and the 5-year cost of ownership. Al-Alawi and Bradley (2013) constructed an ownership cost model of plug-in hybrid electric vehicles.

We have also found some related works that construct joint probability distributions with partial information. Chow and Liu (1968) present the notion of tree dependence to approximate first-order probability distributions. Ku and Kullback (1969) estimated an n-dimensional discrete probability distribution using marginal distributions by using minimum discrimination information estimation method. Keefer (2004) presents a model for approximating probabilistic dependence among binary events. Abbas (2006) explores to use maximum entropy principle to estimate joint distributions from its lower-order assessments. Clemen and Reilly (1999) use copulas to construct joint distributions based on lower-order assessments. Wang and Dyer (2012) estimate multidimensional distributions through the use of copula-based decision trees.

Model and method: true cost to own an automobile

In this section, we discuss the decision model, its variables, and the true cost to own formulation for a car. We use a database of sedan cars of all brands including 32 different brands of car manufacturers with more than 250 different models. We assume that the prices of all the cars are the base price of a car which is the cost of the new car without options, but including standard equipment and factory warranty. We also assume that there are no significant differences in tastes across each alternative car for the decision maker because in real-life situations, decision makers (car buyers) may prefer one brand over another due to the brand loyalty, past reputation, quality, etc.

Model variables

We especially focus on the cost of a car to the customer in 5-year period. There are three uncertainties in our example: fuel economy (mpg), reliability cost, and resale price of the car or depreciation cost of a car. There are also two more uncertainties, the electricity price and the gas price, which helped to calculate the fuel cost of a car. Moreover, safety is another important variable while purchasing automobile. All modern cars have a lot of safety gear, such as seat belts, airbags, antilock brakes, and electronic stability control. In this study, we focus on brand new cars and the prices of safety features are integrated into the buying price of the car. Most of the safety features are integrated into different car make and models, and we take into account all of the automobile alternatives in our analysis. So, safety cost is indirectly integrated into the analysis.

Reliability cost (maintenance and repair)

Reliability cost includes two types of cost: maintenance and repair. Maintenance costs are divided into two: scheduled and unscheduled. Repair cost will be unexpected repair expenses while you are driving in 5-year period.

Fuel economy – MPG

The MPG includes the city and highway mileage. For those vehicles that do not use liquid fuels such as electric vehicles, we use mile per gallon of gasoline-equivalent (MPGe).

Resale value of the car

Depreciation represents the amount of value a vehicle loses each year following purchase.

Insurance

This is the estimated average annual insurance premium. The premium has been determined based on coverage from a major national insurer. Insurance costs assume a driver with a clean record, using the vehicle for personal use.

Electricity price

Electricity price may vary significantly from state to state; therefore, we use the average electricity price paid in cents per kilowatt-hour (kWh) in the United States.

Gasoline price

Gas prices may also vary significantly from state to state. So, in our analysis we use the average pump prices in dollars per gallon of gas in the United States (EIA, 2013).

The following table (Table 1) shows the marginal distributions of four variables. We use our car database and assume that all the marginal distributions are from known families.

The variables 'insurance cost' and 'repair cost' are scaled beta distributions, and the variable 'depreciation percentage' is normally distributed. For gas prices, we use the EIA's short-term energy forecast in our analysis. The parameters for each uncertainty are presented in Table 1.

We discretized each variable by using the McNamee and Celona (1990)'s discretization method. We especially use the shortcut of the McNamee–Celona which is called equal areas method. This method divides the cumulative distribution function into intervals between the P_{100} and the P_{75}, the P_{75} and the P_{25},

Table 1. Marginal distributions of variables of car-buying decision.

Variable	Distribution	Parameters	Range
Insurance cost	Scaled beta	$\alpha = 2, \beta = 15$	[$5k,$13k]
Depreciation (%)	Truncated normal	$\mu = 33.5\%, \sigma = 6\%$	[0%,100%]
Gas prices	Approximated by EIA's short-term energy outlook		
Repair cost	Scaled beta	$\alpha = 2, \beta = 15$	[$500,$3200]

EIA = Energy Information Administration

and the P_{25} and the P_0. This produces a weighting of 0.25, 0.50, and 0.25, respectively. This method weights the 10th (P_{10}), 50th (P_{50}), and 90th (P_{90}) percentiles of probability distribution by 0.250, 0.500, and 0.250, respectively. 10% (low), 50% (base), and 90% (high) percentiles for each uncertainty by using equal areas method are given in Table 2.

We also show the variables in a decision tree format with their three different outcomes using the low, base, and high percentiles of each variable in Figure 1.

Formulation

The decision diagram for the decision situation for true cost of ownership is shown in Figure 2. We use the following formulation to calculate the real cost to own a brand-new car for the first 5 years after purchase.

Table 2. Low, base and high percentiles of variables of car-buying decision.

	Percentiles		
Variable	Low (10%)	Base (50%)	High (90%)
Insurance cost	\$5630	\$6828	\$8620
Depreciation (%)	25.8%	33.5%	41.2%
Gas prices	\$3.40	\$3.53	\$3.87
Repair cost	\$591	\$777	\$1100

Cost to own a car = MRSP of the car + reliability cost (maintenance and repair cost) + fuel cost + insurance cost – resale value of the car

Approximation methods of joint probability distributions

Basic notations and definitions

This section presents the basic notation and definitions that will be used in the remaining sections of the paper. Let

$$F_x(x) = P(X \leq x) \tag{1}$$

be the marginal cumulative distribution function of the random variableX, and let

$$F(x, y) = P(X \leq x, Y \leq y)$$

be the bivariate cumulative distribution function of random variables X and Y.

Define a marginal survival function for variable X as

$$S_x(x) = 1 - F_x(x) = P(X > x) \tag{3}$$

and a bivariate survival function for random variables X and Y as

$$S(x, y) = P(X > x, Y > y) = 1 - F_x(x) - F_y(y) + F(x, y) \tag{4}$$

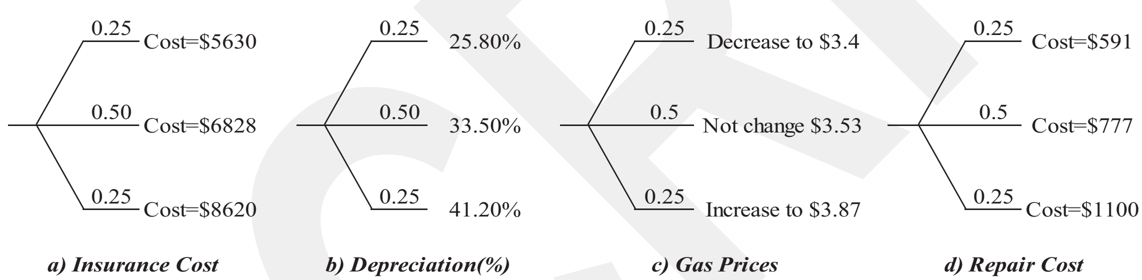

Figure 1. Decision trees of four discretized variables of car-buying decision.

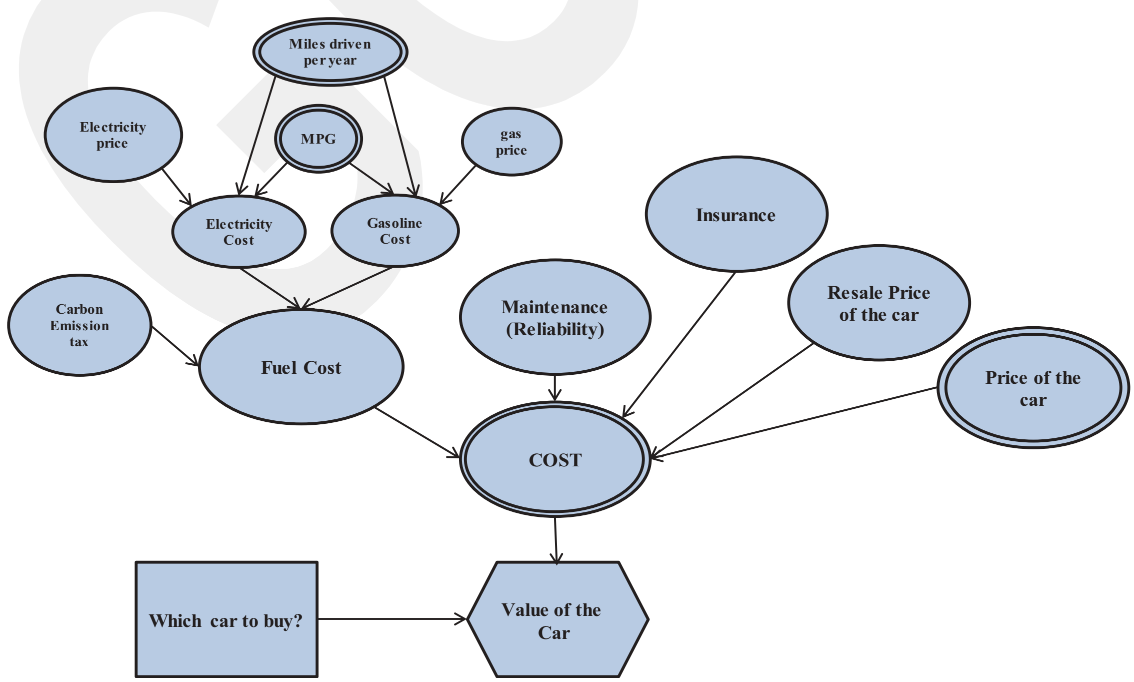

Figure 2. Decision diagram for the decision of car-buying decision situation.

The concept of entropy was introduced in thermodynamics by Rudolf Clausius, where it was used to provide a statement of the second law of thermodynamics. Later, statistical mechanics provided a connection between thermodynamic entropy and the logarithm of the number of microstates in a macrostate of the system. Then, Shannon (1948) used entropy to measure the information and defined the entropy measure using a probability mass function as

$$H(X) = -\sum_{i=1}^{n} p_i \log p_i \tag{5}$$

where p_i is the probability of outcome i.

After several decades of Shannon's entropy definition, Rao et al. (2004) proposed an alternative entropy measure, called as CRE, $\mathcal{E}(S(x))$, using cumulative survival functions as

$$\mathcal{E}(S(x)) = -\int_{0}^{\infty} S(x)\log(S(x))dx \tag{6}$$

where $\mathcal{E}$ is the CRE of the variable X.

After Shannon's entropy explanation, Jaynes (1957) introduced the principle of maximum entropy which maximizes Shannon's entropy subject to certain constraints representing the partial information. The maximum entropy probability mass function for a discrete variable X, having n outcomes, with given constraints is

$$p(x)_{\text{maxent}} = \arg\max -\sum_{i=1}^{n} p(x_i)\log(p(x_i))$$

s.t.

$$\sum_{i=1}^{n} h_i(x)p(x) = \mu_i \tag{7}$$

$$\sum_{i=1}^{n} p(x) = 1$$

$$p(x) \geq 0$$

where $h_i(x)$ are general functions of x and μ_i are given constants. The probability mass function has the maximum entropy when the probability density function has the form $p^*(x) = e^{-\alpha_0-\alpha_1 h_1(x)-\alpha_2 h_2(x)-\ldots-\alpha_n h_n(x)-1}$ where α_i are the Lagrange multipliers for the given moment constraints.

Rao also shows (Rao 2005) how maximum CRE is calculated in similar way to maximum entropy. Let X be a non-negative random variable, and $r_1, \cdots, r_n$ are indicator function or moment constraints, then maximum CRE is defined as

$$\bar{H}^*(x) = \arg\max -\sum_{i=1}^{n} S(x_i)\log(S(x_i))$$

s.t.

$$\sum_{i=1}^{n} r_i(x)S_i(x) = \alpha_i \tag{8}$$

$$x_i, S(x_i) \geq 0$$

where $r_i(x)$ are general functions of x and α_i are given constants. The CRE distribution that maximize the entropy with given constraints has the form as $S^*(x) = \exp\left(-1 - \sum_{i=1}^{n} \lambda_i r_i(x)\right)$ where λ_i are the Lagrange multipliers for the given moment constraints.

In this paper, we use CRE instead of Shannon's traditional entropy measure because the traditional entropy uses probability mass functions or density functions in entropy formulation. On the other hand, CRE uses cumulative distribution functions in both discrete and continuous cases. Moreover, Shannon's entropy has several concerns: (i) it is only defined for distributions with densities, (ii) the entropy of a discrete distribution is always positive, while the differential entropy of a continuous variable may take any value on the extended real line, and (iii) it is inconsistent in the sense that the differential entropy of a uniform distribution in an interval of length a is $\log(a)$, which is zero if $a = 1$, negative if $a < 1$, and positive if $a > 1$.

CRE overcomes some of the problems mentioned above, while keeping many of the important properties of Shannon entropy. Moreover, CRE preserves the well-established principle that the logarithm of the probability of an event should represent the information content in the event and possesses more general mathematical properties than the Shannon entropy. The main advantages of using CRE are: (i) it is always non-negative in both discrete and continuous cases, (ii) it has consistent definitions in both the continuous and discrete domains, (iii) it uses cumulative functions which are more regular than the density functions, because the density is computed as the derivative of the cumulative functions, and (iv) it can be easily computed from the sample data but eliciting density functions is a difficult task.

Maximum CRE

In this section, we apply maximum CRE formulation to approximate joint probability distributions using lower-order assessments. In this paper, we explore to use CRE to approximate joint probability distributions from its lower-order assessments like pairwise, three-way assessments. The available information for a three-variable decision problem should be marginal and/or pairwise assessments. For instance, maximum CRE formulation of a three-variate joint distribution given the pairwise assessments are

$$S^*(x_{1i}, x_{2j}, x_{3k}) = \underset{S(x_{1i},x_{2j},x_{3k})}{\arg\max} -\sum_{x_{1i},x_{2j},x_{3k}} S(x_{1i}, x_{2j}, x_{3k})\ln(S(x_{1i}, x_{2j}, x_{3k}))$$

s.t.

$$\sum_{x_{1i},x_{2j}} S(x_{1i}, x_{2j}, x_{3k}) = S(x_{1i}, x_{2j})$$

$$\sum_{x_{1i},x_{3k}} S(x_{1i}, x_{2j}, x_{3k}) = S(x_{1i}, x_{3k})$$

$$\sum_{x_{2j},x_{3k}} S(x_{1i}, x_{2j}, x_{3k}) = S(x_{2j}, x_{3k})$$

$$S(x_{1i}, x_{2j}, x_{3k}) \geq 0 \tag{9}$$

where x_{1i} refers to ith outcome of the first variable, and $S(x_{1i}, x_{2j}, x_{3k})$refers to the tri-variate joint survival function of the ith outcome of the first variable, jth outcome of the second variable, and k^{th} outcome of the third variable. The solution to the maximum CRE for three variables using bivariate joint assessments is $S^*(x) = e^{-1-\lambda_{ij.}-\lambda_{i.k}-\lambda_{.jk}}$.

First-order dependence trees

In Chow–Liu's first-order dependence tree approach, the mutual information between each two variables is calculated. Chow and Liu (1968) show that a probability distribution of first-order dependence tree structure is the best approximation to the true distribution with respect to the Kullback-Leibler (KL)-divergence measure if its dependence tree has the maximum sum of mutual information pairs from all such first-order dependence trees. The mutual information is defined as

$$\mathrm{MI}(X;Y) = \sum_{x \in X} \sum_{y \in Y} p(x,y) \log\left(\frac{p(x,y)}{p_x(x)p_y(y)}\right) \quad (10)$$

where $p(x,y)$ is the bivariate probability distribution function of variable X and Y, and $p_x(x)$ and $p_y(y)$ are the marginal probability distributions of variable X and Y, respectively.

Chow and Liu provide a simple algorithm for constructing the optimal tree and determine which conditional probabilities are to be used in the product approximation. The method is based on evaluating the mutual information pairs of variables at each stage of the procedure. So, the algorithm simply adds the maximum mutual information pairs to the tree.

Sutcu and Abbas (2015) and Sutcu (2015) showed that the first-order dependence tree approximation is an optimum first-order tree approximation of the joint distribution with respect to the cumulative residual KL-divergence if its dependence tree has the maximum sum of cumulative residual mutual information pairs. Sutcu (2015) defined the mutual information for CRE as

$$\mathrm{MI}_{\mathrm{CRE}} = \left| \sum_{x \in X} \sum_{y \in Y} S(x,y) \left[\log\left(\frac{S(x,y)}{S_x(x)S_y(y)}\right)\right]\right| \quad (11)$$

where $S(x,y)$ is the bivariate survival function of variable X and Y, and $S_x(x)$ and $S_y(y)$ are the marginal survival functions of variable X and Y, respectively.

Results of the analysis

We run a simulation to find the total cost of the midsize sedans after driving it 5 years. In the decision model, we have four different variables and each is discretized to three different values ($3 \times 3 \times 3 \times 3$ joint distribution).

The flowchart for the simulation steps is shown in Figure 3. First, $3 \times 3 \times 3 \times 3$ joint distributions are generated. Then the flowchart follows the steps for first-order dependence tree or maximum entropy approximation methods.

Figure 4 shows the results of the simulation. From the figure, we can easily see that although the price of the Mazda-6 is much cheaper than Honda Accord (cheap more than \$2000), the true cost to own a Mazda-6 is more expensive than the true cost to own a Honda Accord (expensive more than \$5000).

We also run another simulation to see the performance of approximate distributions. We run a simulation with Toyota

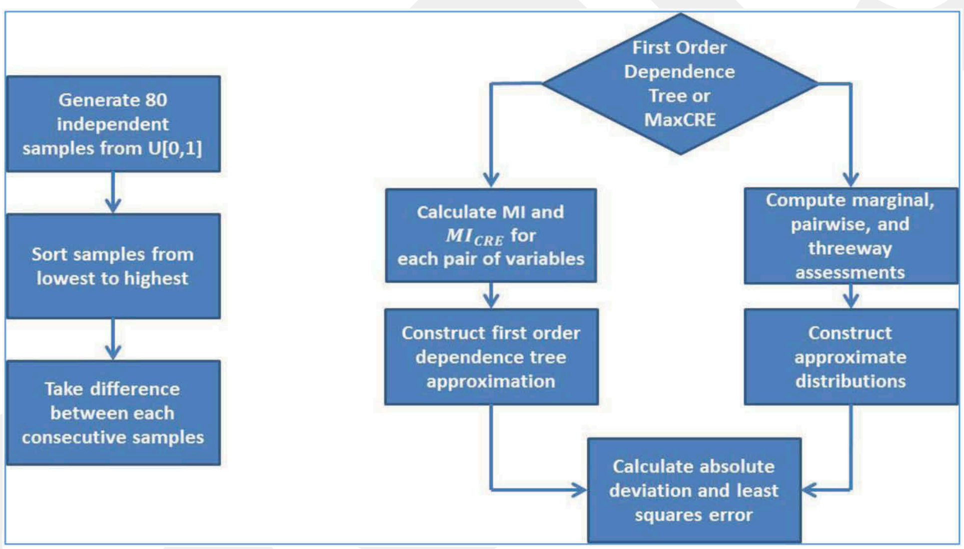

Figure 3. Simulation steps of finding the total cost of ownership.

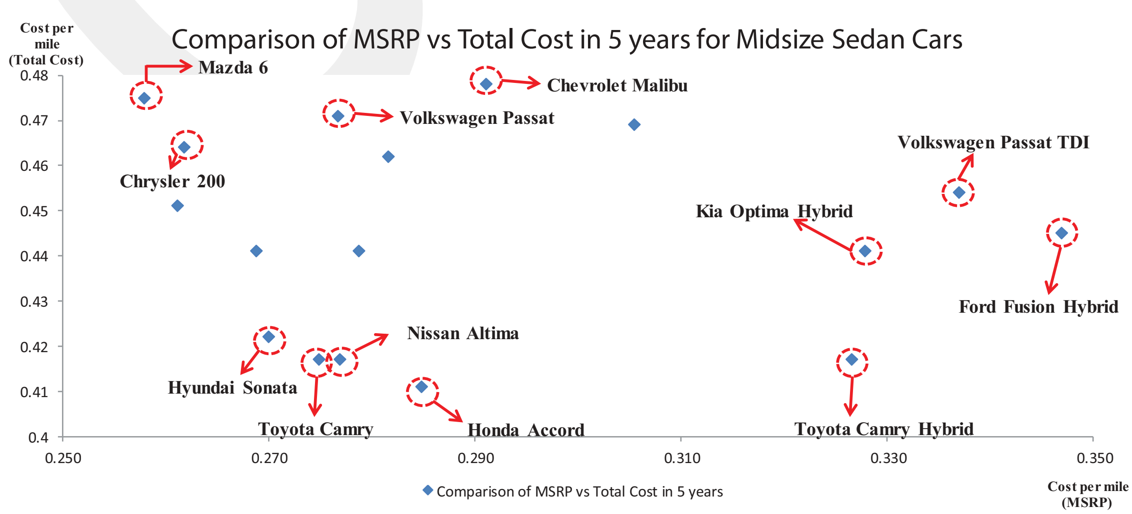

Figure 4. Comparison of selling price of a car and true cost to own a car.

Table 3. Simulation results for Toyota Camry-L Sedan model.

	Maximum entropy		Maximum CRE	
Assessments	Total cost – mean	Total cost – st. dev	Total cost – mean	Total cost – st. dev
True distribution	$31,262		$31,262	
Uniform assessment	$33,349	$0.00	$33,349	$0.00
Marginal assessment	$32,788	$285.41	$32,788	$285.41
Pairwise assessment	$31,300	$268.53	$31,290	$269.17
Three-way assessment	$31,271	$264.32	$31,270	$262.89
First-order dependence tree	$32,344	$272.48	$32,338	$272.21

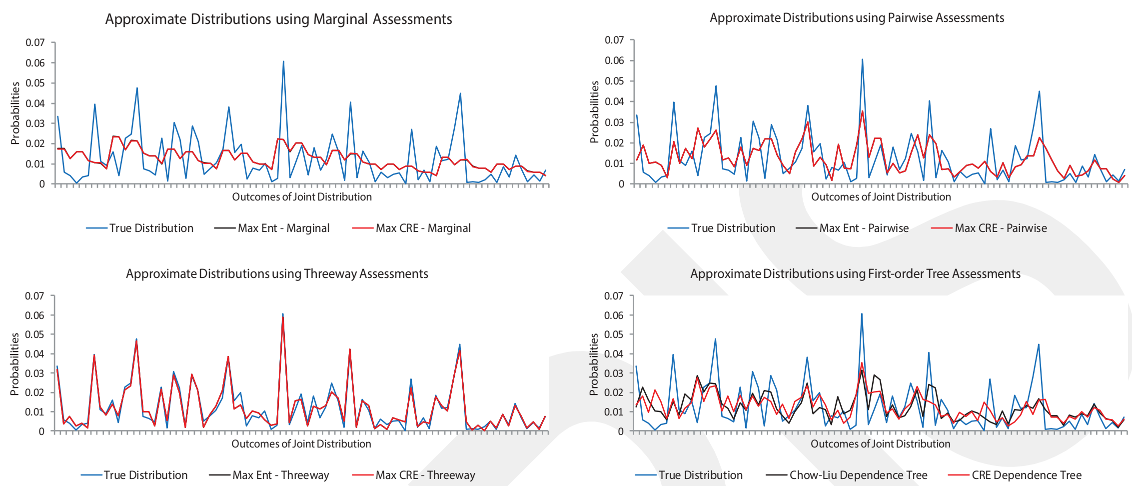

Figure 5. The plots of approximate joint distributions given each lower assessment.

Camry-L Sedan model and used the values of this car for our analysis. We calculated the equal probability, marginal distribution assessment, pairwise assessment, and three-way assessment cases by using maximum entropy and maximum CRE. Also, we run the simulation for both Chow–Liu first-order dependence tree approximation and CRE first-order dependence tree approximation approaches. Table 3 shows the mean value and the standard deviation of the total cost of ownership for all approximation methods.

The results show that the cumulative residual approximation and the Chow–Liu approximations give almost identical accuracy results. The first observation from the simulation results is that CRE method can be used as an alternative method to traditional entropy if cumulative functions, especially survival functions, are present. Therefore, we do not need to calculate the density functions to approximate maximum entropy or first-order dependence trees, and by using CRE method we can directly approximate maximum entropy distribution and first-order dependence trees from cumulative functions.

Figure 5 shows the plots of approximate joint distributions given each lower assessment and the true distribution. Each plot shows the true distribution, maximum entropy, and maximum CRE approximate distributions given different assessments. Another observation from simulation results is that when the dependences among variables are incorporated into the problem, then we can provide more representative joint probability distribution approximations. The value of total cost of ownership gets closer to the true cost if the decision maker provides higher lower-order assessments in a decision situation. Here, if the decision maker only provides marginal assessments, then the maximum CRE and maximum entropy approaches assume independence among variables, and the joint distribution is the multiplication of each marginal distribution. However, if the decision maker provides pairwise assessments which also include the dependencies between variables, then the approximate distribution get closer to the true distribution.

Conclusion

In this paper, we propose a decision model of an automobile's total cost of ownership. Total cost of ownership includes fixed expenses to purchase and own the automobile and variable costs to use and operate the automobile. We also incorporate approximation methods based on CRE to the decision model to construct representative joint probability distributions of a decision maker when only partial information is available. This helps buyers to show that although they can afford to buy a vehicle, they cannot afford to own it. This study showed that although the price of one of the cars is much cheaper than another one, the true cost to own would be more expensive than the true cost to own the other car. This study also showed that CRE can be used as an alternative entropy method when only cumulative probability distributions are present.

Disclosure statement

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References

Abbas, A. E. 2006. "Entropy Methods for Joint Distributions in Decision Analysis." *Engineering Management, IEEE Transactions On* 53 (1): 146–159. doi:10.1109/TEM.2005.861803.

Al-Alawi, B. M., and T. H. Bradley. 2013. “Review of Hybrid, Plug-In Hybrid, and Electric Vehicle Market Modeling Studies.” *Renewable and Sustainable Energy Reviews* 21: 190–203. doi:10.1016/j.rser.2012.12.048.
Barnes, G., and P. Langworthy. 2004. “Per Mile Costs of Operating Automobiles and Trucks.” *Transportation Research Record: Journal of the Transportation Research Board* 1864 (1): 71–77. doi:10.3141/1864-10.
Bordley, R. F. 1982.“A Multiplicative Formula for Aggregating Probability Assessments.” *Management Science* 1137–1148.
Chow, C. K., and C. N. Liu. 1968. “Approximating Discrete Probability Distributions with Dependence Trees.” *IEEE Transactions on Information Theory* IT 14 (3): 462–467. doi:10.1109/TIT.1968.1054142.
Clemen, R. T., G. W. Fischer, and R. L. Winkler. 2000. “Assessing Dependence: Some Experimental Results.” *Management Science* 46 (8): 1100–1115. doi:10.1287/mnsc.46.8.1100.12023.
Clemen, R. T., and T. Reilly. 1999. “Correlations and Copulas for Decision and Risk Analysis.” *Management Science* 45 (2): 208–224. doi:10.1287/mnsc.45.2.208.
Fishburn, P. C., and R. L. Keeney. 1974. “Seven Independence Concepts and Continuous Multiattribute Utility Functions.” *Journal of Mathematical Psychology* 11 (3): 294–327. doi:10.1016/0022-2496(74)90024-8.
Gilmore, E. A., and L. B. Lave. 2013. “Comparing Resale Prices and Total Cost of Ownership for Gasoline, Hybrid and Diesel Passenger Cars and Trucks.” *Transport Policy* 27: 200–208. doi:10.1016/j.tranpol.2012.12.007.
Greenlees, J. S. 1980. “Gasoline Prices and Purchases of New Automobiles.” *Southern Economic Journal* 47 (1): 167–178. doi:10.2307/1057070.
Jaynes, E. T. 1957. “Information Theory and Statistical Mechanics.” *Physical Review* 106 (4): 620. doi:10.1103/PhysRev.106.620.
Jong, G. D., J. Fox, A. Daly, M. Pieters, and R. Smit. 2004. “Comparison of Car Ownership Models.” *Transport Reviews* 24 (4): 379–408. doi:10.1080/0144164032000138733.
Kayser, H. A. 2002. “Gasoline Demand and Car Choice: Estimating Gasoline Demand Using Household Information.” *Energy Economics* 22.3: 331–348.
Keefer, D. 2004. “The Underlying Event Model for Approximating Probabilistic Dependence among Binary Events.” *IEEE Transactions Engineering Manage* 51 (2): 173–182. doi:10.1109/TEM.2004.826014.
Keeney, R. L. 1973. “A Decision Analysis with Multiple Objectives: The Mexico City Airport.” *The Bell Journal of Economics and Management Science* 101–117.
Kelly, D. L. 2007. “Using Copulas to Model Dependence in Simulation Risk Assessment.” *ASME International Mechanical Engineering Congress and Exposition* IMECE2007-41284: 81–89.
Ku, H. H., and S. Kullback. 1969. “Approximating Discrete Probability Distributions.” *IEEE Transations on Information Theory* IT–15 (4): 444–447. doi:10.1109/TIT.1969.1054336.
McNamee, P., and J. Celona. 1990. *Decision Analysis with Supertree*. South San Francisco.
Rao, M. 2005. “More on a New Concept of Entropy and Information.” *Journal of Theoretical Probability* 18 (4): 967–981. doi:10.1007/s10959-005-7541-3.
Rao, M., Y. Chen, B. C. Vemuri, and F. Wang. 2004. “Cumulative Residual Entropy: A New Measure of Information.” *IEEE Transactions Informatics Theory* 50 (6): 1220–1228. doi:10.1109/TIT.2004.828057.
Shannon, C. E. 1948. “A Mathematical Theory of Communication.” *Bell Systems Technical Journal* 27: 379–423 and 623–656. doi:10.1002/j.1538-7305.1948.tb01338.x.
Sutcu, M. “Approximating Multivariate Distributions with Cumulative Residual Entropy: A Study on Dynamic Integrated Climate-Economy Model”, PhD Dissertation, University of Illinois Urbana-Champaign (2015)
Sutcu, M., and A. E. Abbas. 2015. “First Order Dependence Trees with Cumulative Residual Entropy.” *AIP Conference Proceedings* 1641 (1): 512–521.
U.S. Energy Information Administration (EIA) Accessed 22 October 2013. http://www.eia.gov
Van Vliet, O. P., T. Kruithof, W. C. Turkenburg, and A. P. Faaij. 2010. “Techno-Economic Comparison of Series Hybrid, Plug-In Hybrid, Fuel Cell and Regular Cars.” *Journal of Power Sources* 195 (19): 6570–6585. doi:10.1016/j.jpowsour.2010.04.077.
Wang, T., and J. S. Dyer. 2012. “A Copulas-Based Approach to Modeling Dependence in Decision Trees.” *Operations Research* 60 (1): 225–242. doi:10.1287/opre.1110.1004.