



Engineering Computations

Solution of jamming transition problem using adomian decomposition method

Erman Şentürk, Safa Bozkurt Coşkun, Mehmet Tarık Atay,

Article information:

To cite this document:

Erman Şentürk, Safa Bozkurt Coşkun, Mehmet Tarık Atay, (2018) "Solution of jamming transition problem using adomian decomposition method", Engineering Computations, Vol. 35 Issue: 5, pp.1950-1964, <https://doi.org/10.1108/EC-12-2016-0437>

Permanent link to this document:

<https://doi.org/10.1108/EC-12-2016-0437>

Downloaded on: 07 September 2018, At: 17:50 (PT)

References: this document contains references to 56 other documents.

To copy this document: permissions@emeraldinsight.com

The fulltext of this document has been downloaded 9 times since 2018*

Access to this document was granted through an Emerald subscription provided by emerald-srm:380143 []

For Authors

If you would like to write for this, or any other Emerald publication, then please use our Emerald for Authors service information about how to choose which publication to write for and submission guidelines are available for all. Please visit www.emeraldinsight.com/authors for more information.

About Emerald www.emeraldinsight.com

Emerald is a global publisher linking research and practice to the benefit of society. The company manages a portfolio of more than 290 journals and over 2,350 books and book series volumes, as well as providing an extensive range of online products and additional customer resources and services.

Emerald is both COUNTER 4 and TRANSFER compliant. The organization is a partner of the Committee on Publication Ethics (COPE) and also works with Portico and the LOCKSS initiative for digital archive preservation.

*Related content and download information correct at time of download.

Solution of jamming transition problem using adomian decomposition method

Erman Şentürk

Department of Surveying Engineering, Kocaeli University, İzmit, Turkey

Safa Bozkurt Coşkun

Department of Civil Engineering, Kocaeli University, İzmit, Turkey, and

Mehmet Tarık Atay

Department of Mechanical Engineering, Abdullah Gül University, Kayseri, Turkey

1950

Received 14 December 2016

Revised 25 December 2017

3 February 2018

Accepted 4 February 2018

Abstract

Purpose – The purpose of the study is to obtain an analytical approximate solution for jamming transition problem (JTP) using Adomian decomposition method (ADM).

Design/methodology/approach – In this study, the jamming transition is presented as a result of spontaneous deviations of headway and velocity that is caused by the acceleration/breaking rate to be higher than the critical value. Dissipative dynamics of traffic flow can be represented within the framework of the Lorenz scheme based on the car-following model in the one-lane highway. Through this paper, an analytical approximation for the solution is calculated via ADM that leads to a solution for headway deviation as a function of time.

Findings – A highly nonlinear differential equation having no exact solution due to JTP is considered and headway deviation is obtained implementing a number of different initial conditions. The results are discussed and compared with the available data in the literature and numerical solutions obtained from a built-in numerical function of the mathematical software used in the study. The advantage of using ADM for the problem is presented in the study and discussed on the basis of the results produced by the applied method.

Originality/value – This is the first study to apply ADM to JTP.

Keywords Adomian decomposition method, Headway deviation, Jamming transition problem, Lorenz system, Traffic flow

Paper type Research paper

1. Introduction

In most of natural and unnatural networks (Nagatani, 1998, 2002), properties of the transported entities such as traffic flow are the primary interest of practical applications. Within this scope, the traffic congestion related to transportation networks is a practical interest and there have been many studies related to traffic congestion in various disciplines. (Carlier *et al.*, 2008; Sánchez-Medina *et al.*, 2010; Bauza and Gozávez, 2013; Lu *et al.*, 2013; Celikoglu, 2013; Zhang *et al.*, 2014; Qu *et al.*, 2015; Khan *et al.*, 2017). There is a largest admissible capacity of a node in a highway network, such that traffic congestion occurs when the volume of traffic flow exceeds this value. Such nodes may be a bottleneck in the



network. The problem of traffic congestion is a multidisciplinary area studied by the experts of various disciplines including mathematics, engineering, urban planning and logistics.

Traffic flow can be modeled using different approaches classified as microscopic, macroscopic or kinetic (Hidas, 2005; Delitala and Tosin, 2007; Geroliminis and Sun, 2011; Li *et al.*, 2013; Garavello and Piccoli, 2017; Puppo *et al.*, 2017). Microscopic models deal with vehicle movements and interactions between vehicles such as free driving, car following and lane changing. However, macroscopic traffic models provide a measure of vehicle interactions in terms of collective variables. First-order models (Lighthill and Whitham, 1955; Richards, 1956) are one of the two common macroscopic models, describing traffic as a continuous function of the vehicle density and traffic velocity in space and time. The other model is the second-order model that includes an additional partial differential equation for traffic flow to adapt to changing conditions (Cremer, 1979; Payne, 1979). In kinetic models, a probability distribution function $f(t, x, V)$ is defined to express the position x of a vehicle at time t with velocity V .

Traffic jam is a microscopic phenomenon for which some researchers have focused on the jamming transition problem (JTP) in its various forms (Nagel, 1994; Nagatani, 1998, 2000, 2002; Ben-Naim and Krapivsky, 1999; Maerivoet and De Moor, 2005; Peng *et al.*, 2012; Gupta and Redhu, 2013, 2014; Xiao *et al.*, 2017). The problem has been considered using thermodynamic (Nagatani, 1998; Ge *et al.*, 2015), hydrodynamic and kinetic theories (Nagel, 1994; Peng *et al.*, 2011), based on the car-following model (Tang *et al.*, 2009; Gupta and Redhu, 2013; Yang *et al.*, 2013; Qian *et al.*, 2017a; Li *et al.*, 2017), Maxell model (Ben-Naim and Krapivsky, 1999) and cellular automaton model (Maerivoet and De Moor, 2005; Qian *et al.*, 2017b; Xiao *et al.*, 2017). There are also some works available transforming JTP into a nonlinear oscillator with a restoring damping term, via the Lorenz system (Ganji *et al.*, 2011, 2012; Han *et al.*, 2013).

There are also a number of numerical techniques for the problem of traffic congestion in traffic flow, often having restrictions for practical cases. Based on the difficulty in dealing with nonlinear problems, the use of a simple and straightforward solution technique with the minimum computing time is a great advantage to provide a solution to the problem. Various exact (He and Abdou, 2007; Ganji *et al.*, 2009), numerical (Anderson *et al.*, 1984) and semi-analytical (Seyed Alizadeh *et al.*, 2008; Barari, *et al.*, 2008; Ganji *et al.*, 2009; Hashemi Kachapi *et al.*, 2009; Joneidi *et al.*, 2009; Kimiaefar *et al.*, 2009; Babaelahi *et al.*, 2010; Momeni *et al.*, 2010) techniques for solving nonlinear oscillatory systems are available in the literature.

In this study, an analytical approximation technique, namely, the Adomian decomposition method (ADM) (Adomian, 1980, 1994), is used to solve JTP. ADM has been successfully used for solving a large class of problems for the past three decades. ADM produces accurate solutions effectively and easily for linear and nonlinear, ordinary and partial and deterministic or stochastic differential equations with analytical continuous approximations converging accurate solutions rapidly. With the implementation of the method, linearization, perturbation or other restrictive techniques do not supply any advantage compared to ADM. This is a great advantage of the method because the assumptions used in the solution techniques may sometimes seriously change the behavior of the solution. ADM is well suited for physical problems and is selected for the solution of JTP in this paper, which is for the first time that the method is used for the solution of the problem. JTP was previously solved using a number of other techniques (Ganji *et al.*, 2011, 2012). Compared to methods producing solutions given at discrete points, ADM provides an analytical approximation, which is a continuous function with respect to the independent parameter for the governing equation. This solution is continuous, differentiable and

integrable in the solution domain. In the following sections, the governing equation of the problem will be described at first and then ADM will be introduced and applied to the problem. After tabulating ADM solutions and comparing them with available solutions in the literature, the results obtained will be discussed in the conclusion part of the manuscript. In addition, after computing the solution, its advantages over the methods used previously will be evaluated in the text and practical implications of the numerical results will also be discussed.

2. Governing equations of jamming transition problem

Lorenz system is proposed for modeling JTP as a nonlinear non-conservative oscillator in terms of a third-order nonlinear differential equation. In this model, the jamming transition is presented as a result of spontaneous deviations of headway and velocity that is caused by the acceleration/braking rate to be higher than the critical value. Dissipative dynamics of traffic flow can be represented within the framework of the Lorenz scheme based on the car-following model in the one-lane highway (Olemskoi and Khomenko, 2001, Khomenko *et al.*, 2004).

Based on this model, the system of equations given below is obtained:

$$\dot{\eta} = -\eta/t_{\eta} + v \tag{1}$$

$$\dot{v} = -v/t_v + g_v \tau \eta \tag{2}$$

$$\dot{\tau} = (\tau_0 - \tau)/t_{\tau} - g_{\tau} \eta v + \lambda(t) \tag{3}$$

In this equation, η is headway deviation, τ is velocity deviation, v is acceleration/braking time, t_{η} , t_v and t_{τ} , appropriate relaxation time, g_v and g_{τ} are positive constants and dot means differentiation with respect to time. The system of equations does not take the fluctuation of acceleration/braking time into account.

First terms on the right-hand side of equations between equations 1 and 3 represent the relaxation of each quantity to an equilibrium value. The second term on the right-hand side of equation (2) provides a positive feedback of headway deviation η and acceleration/braking time τ on velocity deviation v . Due to the positive feedback, velocity deviation will increase and this will be the reason for traffic jam formation. From equation (3), it may be observed that relaxation for acceleration/braking time τ occurs to a finite value τ_0 rather than zero, which represents the time necessary for an automobile to reach a characteristic velocity.

Due to LeChatelier principle, to impede a jam formation, the headway deviation η and its velocity deviation v should vary to prevent the growth of acceleration/braking time τ , since the decrease of acceleration/braking time assists to the formation of stable traffic flow (Khomenko *et al.*, 2004).

There is no analytical solution available for the system of equations. To obtain an acceptable solution, some simple assumptions are introduced such that $t_{\eta} \gg t_{\tau}$ and $t_{\eta} \approx t_v$, i.e. relaxation time for acceleration/braking is very small when compared to relaxation times for headway deviation and its velocity. Due to these conditions, coordination of acceleration/braking time τ occurs by the variation of headway and velocity deviations. The assumptions lead to $\dot{\tau}t_{\tau} \approx 0$ and equation (3) becomes,

$$\tau = \tau_0 - g_\tau t_\tau \eta v \tag{4}$$

Introducing t_η , $\eta_m = (g_v g_\tau t_\tau t_\eta)^{-1/2}$, $v_m = t_\eta^{-3/2} (g_v g_\tau t_\tau)^{-1/2}$, $\tau_c = (g_v t_\eta^2)^{-1}$ as natural scale factors for time, headway deviation, velocity deviation and acceleration/braking time, respectively, and $g_v t_\tau t_\eta^2$ for noise term of the characteristic acceleration/braking time τ_0 . Velocity deviation and its time derivative can be obtained from equation (1). Inserting these parameters and equation (4) into equation (2) provides the following scaled nonlinear stochastic oscillation equation (Khomenko *et al.*, 2004),

$$\ddot{\eta} + \dot{\eta}(1 + \sigma + \eta^2) - \eta(\varepsilon - \sigma) + \eta^3 = 0 \tag{5}$$

where $\sigma \equiv t_\eta/t_v$ and $\varepsilon \equiv \tau_0/\tau_c$. It is not possible to produce an analytical solution for equation (5) in which highly nonlinear terms exist. In such problems, approximate solution techniques must be used to provide a solution. Such methods may be numerical, variational or analytical approximate methods. In this study, an analytical approximate solution technique, i.e. ADM, is used to obtain an analytical approximation for JTP.

3. Adomian decomposition method

In the ADM, a differential equation of the following form is considered:

$$Lu + Ru + Nu = g(x) \tag{6}$$

where L is the linear operator which is highest-order derivative, R is the remainder of linear operator including derivatives of less order than L , Nu represents the nonlinear terms and g is the source term. Equation (6) can be rearranged as:

$$Lu = g(x) - Ru - Nu \tag{7}$$

The inverse operator L^{-1} is applied to both sides of equation (7) to obtain:

$$u = L^{-1}(g(x)) - L^{-1}(Ru) - L^{-1}(Nu) \tag{8}$$

After integrating source term $g(x)$ and combining it with the terms arising from boundary conditions of the problem, a function $f(x)$ is defined in equation (8) as follows:

$$u = f(x) - L^{-1}(Ru) - L^{-1}(Nu) \tag{9}$$

The nonlinear operator $Nu = F(u)$ is represented by an infinite series of specially generated (Adomian) polynomials for the specific nonlinearity. Assuming Nu is analytic, we write:

$$F(u) = \sum_{k=0}^{\infty} A_k \tag{10}$$

The polynomials A_k 's are generated for all kinds of nonlinearity so that they depend only on u_0 to u_k components. These polynomials can be produced by the following algorithm:

$$A_0 = F(u_0) \tag{11}$$

$$A_1 = u_1 F'(u_0) \tag{12}$$

$$A_2 = u_2 F'(u_0) + \frac{1}{2!} u_1^2 F''(u_0) \tag{13}$$

$$A_n = \sum_{v=1}^n c(v, n) F^{(v)}(u_0) \tag{14}$$

where $c(v, n)$ are products of v components of u 's whose subscripts sum to n -divided by the factorial of the number of repeated subscripts. Detailed information about the method can be found in the literature (Adomian, 1980, 1994). The solution of the problem, $u(x)$, is defined by the following series:

$$u = \sum_{k=0}^{\infty} u_k \tag{15}$$

where the components of the series are determined recursively as follows:

$$u_0 = f(x) \tag{16}$$

$$u_{k+1} = -L^{-1}(Ru_k) - L^{-1}(A_k), \quad k \geq 0 \tag{17}$$

4. Application of Adomian decomposition method to jamming transition problem

ADM is applied to the problem by selecting the second derivative term for the linear operator in the formulation. Then, equation (18) is obtained:

$$\ddot{\eta} = -\left[(1 + \sigma)\dot{\eta} + (\sigma - \varepsilon)\eta + \dot{\eta}\eta^2 + \eta^3 \right] \tag{18}$$

Operator $L_{tt} = d^2/dt^2$ leads to its inverse operator as, $L_{tt}^{-1} = \int_0^t \int_0^t dt dt$. The inverse operator is then applied to both sides of equation (18):

$$L_{tt}^{-1} L_{tt} \eta = -L_{tt}^{-1} \left[(1 + \sigma)\dot{\eta} + (\sigma - \varepsilon)\eta + \dot{\eta}\eta^2 + \eta^3 \right] \tag{19}$$

Initial conditions for the problem are $\eta(0) = A$ and $\dot{\eta}(0) = 0$. These conditions are now introduced to formulation as below:

$$\eta - \eta(0) - t\dot{\eta}(0) = -L_{tt}^{-1} \left[(1 + \sigma)\dot{\eta} + (\sigma - \varepsilon)\eta + \dot{\eta}\eta^2 + \eta^3 \right] \tag{20}$$

$$\eta = A - L_{tt}^{-1} \left[(1 + \sigma)\dot{\eta} + (\sigma - \varepsilon)\eta + \dot{\eta}\eta^2 + \eta^3 \right] \tag{21}$$

Equation (21) is now decomposed according to its linear and nonlinear terms. To treat the nonlinear terms, Adomian polynomials should be used:

$$\eta = A - L_{tt}^{-1}[(1 + \sigma)\dot{\eta} + (\sigma - \varepsilon)\eta] - L_{tt}^{-1}[\dot{\eta}\eta^2] - L_{tt}^{-1}[\eta^3] \quad (22)$$

Hence, the solution is arranged as follows by introducing Adomian polynomials for the nonlinear terms in equation (22):

$$\sum_{n=0}^{\infty} \eta_n(t) = \eta_0(t) + L_{tt}^{-1}\left(\sum_{n=0}^{\infty} A_n\right) + L_{tt}^{-1}\left(\sum_{n=0}^{\infty} B_n\right) \quad (23)$$

Initial approximation for the solution procedure is selected from equation (22) which produces a recurrence relation based on initial approximation as below:

$$\eta_0(t) = A, \quad (24)$$

$$\eta_{n+1}(t) = -L_{tt}^{-1}[(1 + \sigma)\dot{\eta}_n + (\sigma - \varepsilon)\eta_n] - L_{tt}^{-1}\left(\sum_{n=0}^{\infty} A_n\right) - L_{tt}^{-1}\left(\sum_{n=0}^{\infty} B_n\right) \quad (25)$$

Adomian polynomials A_n and B_n in equation (25) for the first five approximations are as follows:

$$A_0 = \eta_0^2 \eta_0' \quad (26)$$

$$A_1 = \eta_0(2\eta_1 \eta_0' + \eta_0 \eta_1') \quad (27)$$

$$A_2 = (\eta_0' \eta_1^2 + 2\eta_0 \eta_1' \eta_1 + \eta_0(2\eta_2 \eta_0' + \eta_0 \eta_2')) \quad (28)$$

$$A_3 = (\eta_1' \eta_1^2 + 2(\eta_2 \eta_0' + \eta_0 \eta_2') \eta_1 + \eta_0(2\eta_3 \eta_0' + 2\eta_2 \eta_1' + \eta_0 \eta_3')) \quad (29)$$

$$A_4 = (\eta_2' \eta_1^2 + 2(\eta_3 \eta_0' + \eta_0 \eta_3') \eta_1 + \eta_2^2 \eta_0' + 2\eta_2(\eta_1 \eta_1' + \eta_0 \eta_2') + \eta_0(2\eta_4 \eta_0' + 2\eta_3 \eta_1' + \eta_0 \eta_4')) \quad (30)$$

$$B_0 = \eta_0^3 \quad (31)$$

$$B_1 = 3\eta_0^2 \eta_1 \quad (32)$$

$$B_2 = 3\eta_0(\eta_1^2 + \eta_0 \eta_2) \quad (33)$$

$$B_3 = (\eta_1^3 + 6\eta_0\eta_2\eta_1 + 3\eta_0^2\eta_3) \tag{34}$$

$$B_4 = 3(\eta_2\eta_1^2 + 2\eta_0\eta_3\eta_1 + \eta_0(\eta_2^2 + \eta_0\eta_4)) \tag{35}$$

1956

The algorithm to produce Adomian polynomials has been given in equations 11-14. However, the reader may refer (Babolian and Javadi, 2004, Zhu *et al.*, 2005, Pourdavish, 2006) to get additional information on various techniques for calculating these polynomials. After conducting successive approximations with the expression given in equation (25), the headway deviation η can be obtained through ADM as an analytical expression, which is a continuous function in the solution domain.

5. Results and discussion

As case studies of the problem, previously available solutions are going to be used for comparison purposes. For this purpose, some numerical values are preselected from previous studies (Ganji *et al.*, 2011, 2012). Each set of different values of parameters is called a mode. Table I includes values for four modes to be used in numerical applications.

Tables II-V present solutions for each mode given in Table I, respectively. ADM solution procedure is conducted up to five iterations and a comparison with the numerical solutions of the same nonlinear differential equation is provided in each table. Both ADM and numerical solutions are obtained by Wolfram Mathematica Software. Solutions for each mode are obtained for different values of amplitude changing in the interval [0, 1] with a step size of 0.1. In Figures 1-4, absolute errors and total absolute errors are given for each mode. In the left part of the figures, absolute errors obtained from iterations of ADM are presented. As it can be seen, proposed technique introduces less error for fifth iteration solution. In the right part of the figures, cumulative absolute errors are given for all amplitudes.

Table I.
Different values for parameters ε , σ and t

Mode	ε	σ	t
1	0.25	0.75	0.25
2	0.75	2.50	0.50
3	3.25	0.75	0.75
4	2.00	0.75	1.00

Table II.
Comparisons for ADM solution and numerical solution for Mode 1

A	Numerical	ADM _{n=1}	ADM _{n=2}	ADM _{n=3}	ADM _{n=4}	ADM _{n=5}
0.1	0.098620	0.098406	0.098644	0.098618	0.098620	0.098620
0.2	0.197086	0.196625	0.197139	0.197081	0.197086	0.197086
0.3	0.295246	0.294469	0.295339	0.295237	0.295247	0.295246
0.5	0.490073	0.488281	0.490311	0.490047	0.490075	0.490072
0.7	0.682044	0.678344	0.682608	0.681973	0.682052	0.682043
0.9	0.870297	0.863156	0.871579	0.870105	0.870323	0.870294
1.0	0.962824	0.953125	0.964722	0.962511	0.962872	0.962817

Tables and Figures depict that absolute errors decrease when increasing numbers of iteration, i.e. the series in equation (15) is convergent and shows asymptotic behavior. Even with five iterations, ADM results show very good agreement with the numerical results. In Table VI, ADM results are compared with the previously available results (Ganji *et al.*, 2011; Ganji *et al.*, 2012). In Ganji *et al.*'s study (2012), differential transform method (DTM) was used to solve JTP and in Ganji *et al.*'s study (2011), homotopy perturbation method (HPM) and variational iteration method (VIM) were used to solve the same problem. ADM has a simpler algorithm providing an initial approximation with the formulation. The order of ADM approximation is chosen from the available order of solutions in the previous results.

Table VI depicts that ADM solutions show better agreement with the numerical results compared to VIM and DTM. In Ganji *et al.*'s study (2012), DTM approximation was conducted up to nine iterations. In Table VII, ninth-order solutions of ADM and DTM approximations and numerical solutions are compared. Since no solutions existed in Ganji *et al.*'s study (2012) for Mode 1, it was not included in the comparison of ninth-order

A	Numerical	ADM _{n=1}	ADM _{n=2}	ADM _{n=3}	ADM _{n=4}	ADM _{n=5}
0.1	0.087166	0.078000	0.091686	0.085448	0.087699	0.087025
0.2	0.174021	0.155250	0.183396	0.170401	0.175172	0.173705
0.3	0.260266	0.231000	0.275189	0.254350	0.262221	0.259695
0.5	0.429830	0.375000	0.459635	0.417019	0.434574	0.428194
0.7	0.593987	0.504000	0.647488	0.568275	0.605037	0.589361
0.9	0.751472	0.612000	0.843960	0.700680	0.777295	0.738293
1.0	0.827500	0.656250	0.948079	0.756328	0.866980	0.805290

Table III.
Comparisons for ADM solution and numerical solution for Mode 2

A	Numerical	ADM _{n=1}	ADM _{n=2}	ADM _{n=3}	ADM _{n=4}	ADM _{n=5}
0.1	0.152310	0.170031	0.147326	0.153488	0.152071	0.152354
0.2	0.302013	0.338375	0.291890	0.304282	0.301578	0.302097
0.3	0.446731	0.503344	0.431061	0.449929	0.446203	0.446845
0.5	0.713915	0.816406	0.684158	0.718439	0.713895	0.713955
0.7	0.944676	1.095720	0.893222	0.951217	0.946858	0.943885
0.9	1.137780	1.327780	1.055400	1.151770	1.143340	1.133990
1.0	1.221360	1.421880	1.121950	1.243040	1.227550	1.215080

Table IV.
Comparisons for ADM solution and numerical solution for Mode 3

A	Numerical	ADM _{n=1}	ADM _{n=2}	ADM _{n=3}	ADM _{n=4}	ADM _{n=5}
0.1	0.140700	0.162000	0.131930	0.143638	0.139875	0.140900
0.2	0.278053	0.321000	0.260198	0.283898	0.276498	0.278409
0.3	0.409103	0.474000	0.381490	0.417862	0.407003	0.409511
0.5	0.644004	0.750000	0.593750	0.659549	0.641495	0.643856
0.7	0.836798	0.966000	0.762510	0.863280	0.833060	0.835023
0.9	0.989290	1.098000	0.909570	1.027820	0.978607	0.988652
1.0	1.052570	1.125000	0.992188	1.087390	1.038810	1.055300

Table V.
Comparisons for ADM solution and numerical solution for Mode 4

solutions. In Table VII, ADM results are shown in better agreement with the numerical solution especially for larger values of amplitudes.

In Figure 5, the variation of headway deviation with time is depicted and it is compared with the numerical solution. It can be seen that from figures, approximations are getting better with the increasing number of order. In Figure 5(a) and (b), fifth-order approximation is in very good agreement with the numerical solution up to half interval of time, whereas in Figure 5(c) and (d), fifth-order approximations are in very good agreement with the numerical solution for the whole interval of time. These behaviors are discussed in conclusion section.

6. Conclusion

In this study, ADM is used for the analysis of JTP in traffic congestion. The problem is modeled as a nonlinear non-conservative oscillator through Lorenz system. ADM is used for the first time in the solution of JTP. The problem was previously analyzed by using DTM, HPM and VIM. The proposed solution is obtained up to the ninth order of approximation and the results show that ADM provides good approximations even with less number of iterations. This behavior is due to a rapid convergence of the method even for a highly nonlinear differential equation governing the problem. Another advantage of the method is its simple formulation which already provides an initial approximation in the formulation

Figure 1.
The absolute errors and cumulative absolute errors for Mode 1

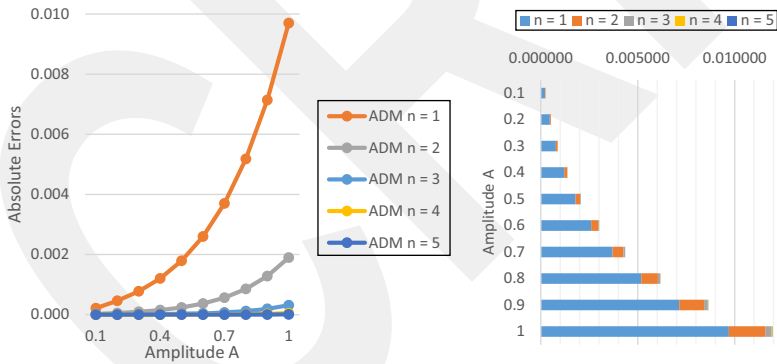
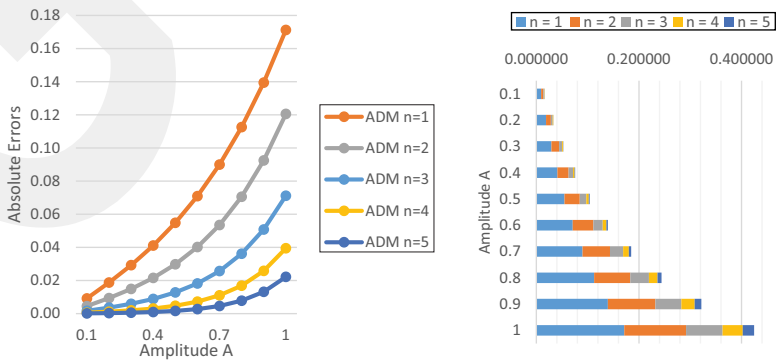


Figure 2.
The absolute errors and cumulative absolute errors for Mode 2



stage. ADM formulates the solution without any transformation of the equation, without any linearization procedure or without perturbation techniques. These properties of the method provide simplicity in the formulation when compared to other techniques.

The analyses showed that ADM with fifth-order approximation provided accurate results with negligible absolute errors. These solutions are in very good agreement with the numerical solution computed by the use of Mathematica software. Previous solutions to the problem obtained by using several methods were used for the comparison purposes. These methods are HPM, VIM and DTM. The results showed that ADM results are better approximations for all cases than VIM and DTM. ADM and HPM solutions with the same order of approximations were found to be very close or the same.

An initial headway deviation is assumed as an initial condition and fifth order of approximation is accurate enough for the solution given in Figure 5(c) and (d). However, additional terms to increase the order of approximation is required for better approximations, which may be observed in Figure 5(a) and (b). This behavior is due to the magnitude and the sign of $\varepsilon - \sigma$. Parameter σ is scaled inverse of relaxation time for velocity deviation and ε is scaled characteristic acceleration/breaking time. When $\varepsilon - \sigma$ is negative, headway deviation decreases with time and with the increase of negative magnitude, higher-order ADM approximations are required for an accurate solution. Negative sign indicates less characteristic acceleration/breaking time and this behavior leads to less congestion in traffic

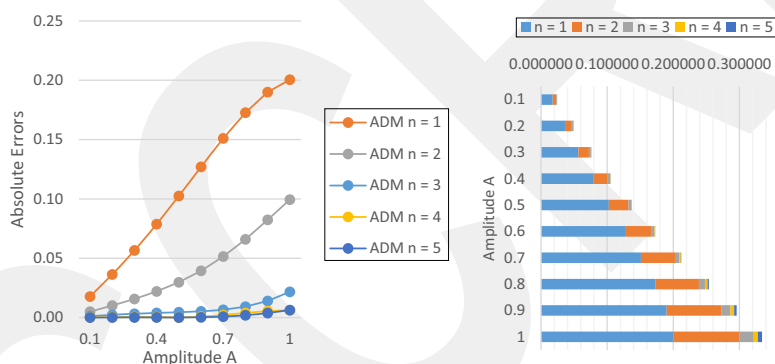


Figure 3.
The absolute errors and cumulative absolute errors for Mode 3

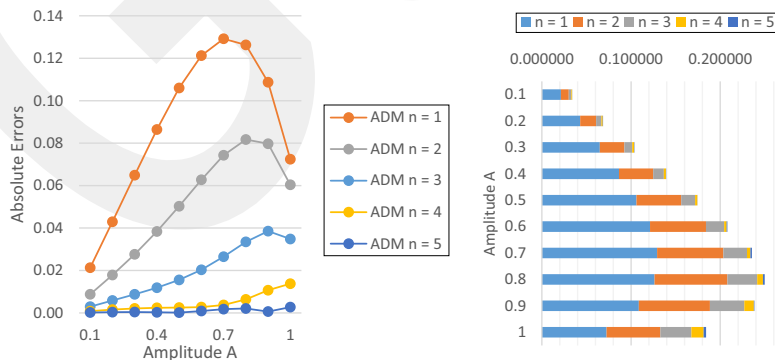


Figure 4.
The absolute errors and cumulative absolute errors for Mode 4

EC
35,5

1960

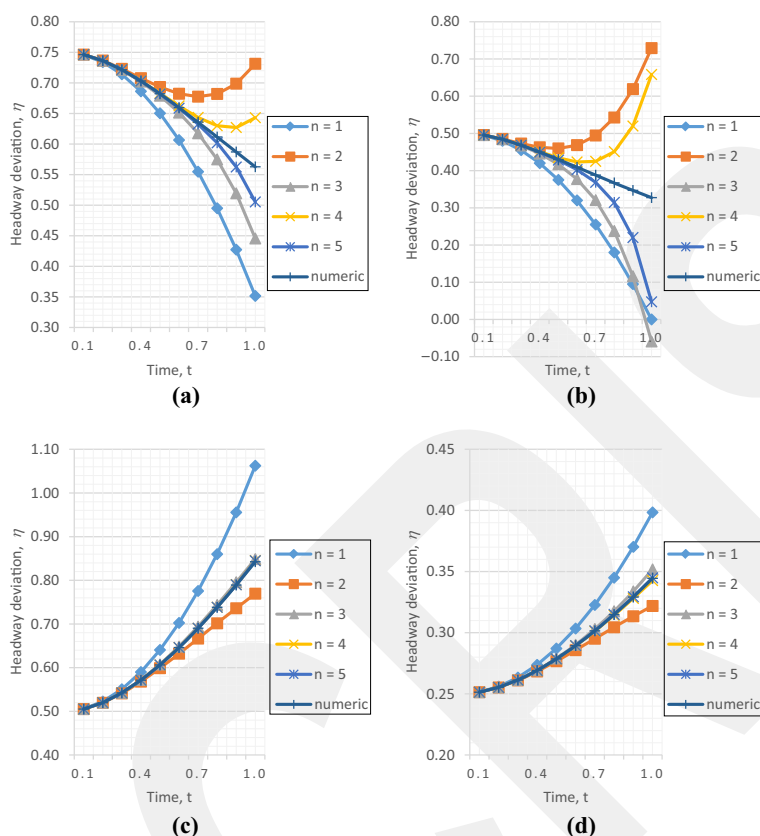
Table VI.
Comparisons for
third-order results of
ADM, DTM, HPM
and VIM solutions

Mode	A	Numerical	ADM _{n=3}	DTM _{n=3}	HPM _{n=3}	VIM _{n=3}	
1	0.1	0.098620	0.098618	0.098406	0.098618	0.098618	
	0.2	0.197086	0.197081	0.196620	0.197081	0.197081	
	0.3	0.295246	0.295237	0.294470	0.295237	0.295237	
	0.5	0.490073	0.490047	0.490230	0.490047	0.490049	
	0.7	0.682044	0.681973	0.678340	0.681973	0.681981	
	0.9	0.870297	0.870105	0.863160	0.870105	0.870135	
	1.0	0.962824	0.962511	0.953120	0.962511	0.962570	
	2	0.1	0.087166	0.087699	0.090870	0.087699	0.085454
		0.2	0.174021	0.175172	0.181653	0.175172	0.170453
		0.3	0.260266	0.262221	0.272285	0.262221	0.245367
0.5		0.429830	0.434574	0.453125	0.434574	0.418078	
0.7		0.593987	0.605037	0.634340	0.605037	0.572092	
0.9		0.751472	0.777295	0.818880	0.777294	0.711870	
1.0		0.827500	0.866980	0.914063	0.866980	0.774554	
3		0.1	0.152310	0.152071	0.139218	0.152071	0.153550
		0.2	0.302013	0.301578	0.276455	0.301578	0.304792
		0.3	0.446731	0.446203	0.409806	0.446203	0.451683
	0.5	0.713915	0.713895	0.658203	0.713895	0.726723	
	0.7	0.944676	0.946858	0.874116	0.946858	0.972742	
	0.9	1.137780	1.143340	1.054001	1.143340	1.189564	
	1.0	1.221360	1.227550	1.131836	1.227550	1.286738	
	4	0.1	0.140700	0.139875	0.125627	0.162000	0.131930
		0.2	0.278053	0.276498	0.248803	0.321000	0.260198
		0.3	0.409103	0.407003	0.367280	0.474000	0.381490
0.5		0.644004	0.641495	0.583333	0.750000	0.593750	
0.7		0.836798	0.833060	0.767387	0.966000	0.762510	
0.9		0.989290	0.978607	0.929040	1.098000	0.909570	
1.0		1.052570	1.038810	1.010417	1.125000	0.992188	

Table VII.
Comparisons for
ADM and DTM
($n = 9$)

Mode	Method	0.1	0.2	0.3	A	0.5	0.7	0.9	1.0
2	ADM	0.087165	0.174019	0.260261	0.429796	0.593764	0.750126	0.827251	
	DTM	0.087164	0.174018	0.260264	0.429839	0.594030	0.751465	0.827707	
3	Num.	0.087166	0.174021	0.260266	0.429830	0.593987	0.751472	0.827500	
	ADM	0.152310	0.302014	0.446734	0.713927	0.944709	1.137950	1.221760	
	DTM	0.152354	0.302168	0.447132	0.715440	0.948418	1.144452	1.229225	
4	Num.	0.152310	0.302013	0.446731	0.713915	0.944676	1.137780	1.221360	
	ADM	0.140700	0.278058	0.409121	0.644096	0.837099	0.989567	1.052440	
	DTM	0.140792	0.278331	0.409730	0.645857	0.839791	0.991466	1.053526	
	Num.	0.140700	0.278053	0.409103	0.644004	0.836798	0.989290	1.052570	

flow. When $\varepsilon - \sigma$ is positive, headway deviation increases with time and with the increase of positive magnitude, accurate results may be obtained with limited number of successive approximations that is equivalent to less order of ADM approximation. Positive sign indicates greater characteristic acceleration/breaking time and this behavior leads to more congestion in traffic flow due to greater characteristic acceleration/breaking time and the increasing headway deviation.



Notes: (a) η - t diagram with $\varepsilon = 0.25$, $\sigma = 0.75$ and $A = 0.75$; (b) η - t diagram with $\varepsilon = 0.75$, $\sigma = 2.50$ and $A = 0.50$; (c) η - t diagram with $\varepsilon = 3.25$, $\sigma = 0.75$ and $A = 0.50$; (d) η - t diagram with $\varepsilon = 2.00$, $\sigma = 0.75$ and $A = 0.25$

Figure 5. Variation of headway deviation with time (fifth order of approximation) and comparison with numerical solution

ADM showed a successful performance in the solution of JTP in traffic congestion and provided an analytical type solution of the problem with an easy formulation and application process.

References

Adomian, G. (1980), *Applied Stochastic Processes*, Academic Press, New York, NY.
 Adomian, G. (1994), *Solving Frontier Problems of Physics: The Decomposition Method*, Springer, Berlin.
 Anderson, D.A., Tannehill, J.C. and Pletcher, R. (1984), *Computational Fluid Mechanics and Heat Transfer*, Hemisphere Publishing Corporation, New York, NY.
 Babaelahi, M., Ganji, D.D. and Joneidi, A.A. (2010), "Analysis of velocity equation of steady flow of a viscous incompressible fluid in channel with porous walls", *International Journal for Numerical Methods in Fluids*, Vol. 63 No. 9, pp. 1048-1059.

- Babolian, E. and Javadi, S. (2004), "New method for calculating Adomian polynomials", *Applied Mathematics and Computation*, Vol. 153 No. 1, pp. 253-259.
- Barari, A., Omidvar, M., Ghotbi, A.R. and Ganji, D.D. (2008), "Application of homotopy perturbation method and variational iteration method to nonlinear oscillator differential equations", *Acta Applicandae Mathematicae*, Vol. 104 No. 2, pp. 161-171.
- Bauza, R. and Gozálviz, J. (2013), "Traffic congestion detection in large-scale scenarios using vehicle-to-vehicle communications", *Journal of Network and Computer Applications*, Vol. 36 No. 5, pp. 1295-1307.
- Ben-Naim, E. and Krapivsky, P.L. (1999), "Maxwell model of traffic flows", *Physical Review E*, Vol. 59 No. 1, pp. 88-97.
- Carlier, G., Jimenez, C. and Santambrogio, F. (2008), "Optimal transportation with traffic congestion and wardrop equilibria", *SIAM Journal on Control and Optimization*, Vol. 47 No. 3, pp. 1330-1350.
- Celikoglu, H.B. (2013), "An approach to dynamic classification of traffic flow patterns", *Computer-Aided Civil and Infrastructure Engineering*, Vol. 28 No. 4, pp. 273-288.
- Cremer, M. (1979), *Der Verkehrsfluss Auf Schnellstraßen*, Springer, Berlin.
- Delitala, M. and Tosin, A. (2007), "Mathematical modeling of vehicular traffic: a discrete kinetic theory approach", *Mathematical Models and Methods in Applied Sciences*, Vol. 17 No 6, pp. 901-932.
- Ganji, S.S., Barari, A., Ibsen, L.B. and Domairry, G. (2012), "Differential transform method for mathematical modeling of jamming transition problem in traffic congestion flow", *Central European Journal of Operations Research*, Vol. 20 No. 1, pp. 87-100.
- Ganji, S.S., Barari, A., Najafi, M. and Domairry, G. (2011), "Analytical evaluation of jamming transition problem", *Canadian Journal of Physics*, Vol. 89 No. 6, pp. 729-738.
- Ganji, S.S., Ganji, D.D., Karimpour, S. and Babazadeh, H. (2009), "Applications of he's homotopy perturbation method to obtain second-order approximations of the coupled two-degree-of-freedom systems", *International Journal of Nonlinear Sciences and Numerical Simulation*, Vol. 10 No. 3, pp. 305-314.
- Ganji, Z.Z., Ganji, D.D. and Bararnia, H. (2009), "Approximate general and explicit solutions of nonlinear BBMB equations by EXP-function method", *Applied Mathematical Modelling*, Vol. 33 No. 4, pp. 1836-1841.
- Garavello, M. and Piccoli, B. (2017), "Boundary coupling of microscopic and first order macroscopic traffic models", doi: [10.1007/s00030-017-0467-5](https://doi.org/10.1007/s00030-017-0467-5).
- Ge, H.X., Zheng, P.J., Wang, W. and Cheng, R.J. (2015), "The car following model considering traffic jerk", *Physica A: Statistical Mechanics and Its Applications*, Vol. 433, pp. 274-278.
- Geroliminis, N. and Sun, J. (2011), "Properties of a well-defined macroscopic fundamental diagram for urban traffic", *Transportation Research Part B: Methodological*, Vol. 45 No. 3, pp. 605-617.
- Gupta, A.K. and Redhu, P. (2013), "Analyses of driver's anticipation effect in sensing relative flux in a new lattice model for two-lane traffic system", *Physica A: Statistical Mechanics and Its Applications*, Vol. 392 No. 22, pp. 5622-5632.
- Gupta, A.K. and Redhu, P. (2014), "Analysis of a modified two-lane lattice model by considering the density difference effect", *Communications in Nonlinear Science and Numerical Simulation*, Vol. 19 No. 5, pp. 1600-1610.
- Han, X.L., Ouyang, C. and Song, T. (2013), "The homotopy analysis method for a class of jamming transition problem in traffic flow", doi: [10.7498/aps.62.170203](https://doi.org/10.7498/aps.62.170203).
- Hashemi, Kachapi, S.H.A., Barari, A., Tolou, N. and Ganji, D.D. (2009), "Solution of strongly nonlinear oscillation systems using variational approach", *Journal of Applied Functional Analysis*, Vol. 4 No. 3, pp. 528-535.

- He, J.H. and Abdou, M.A. (2007), "New periodic solutions for nonlinear evolution equations using exp-function method", *Chaos, Solitons & Fractals*, Vol. 34 No. 5, pp. 1421-1429.
- Hidas, P. (2005), "Modelling vehicle interactions in microscopic simulation of merging and weaving", *Transportation Research Part C: Emerging Technologies*, Vol. 13 No. 1, pp. 37-62.
- Joneidi, A.A., Ganji, D.D. and Babaelahi, M. (2009), "Differential transformation method to determine fin efficiency of convective straight fins with temperature dependent thermal conductivity", *International Communications in Heat and Mass Transfer*, Vol. 36 No. 7, pp. 757-762.
- Khan, S.M., Dey, K.C. and Chowdhury, M. (2017), "Real-time traffic state estimation with connected vehicles", *IEEE Transactions on Intelligent Transportation Systems*, Vol. 18 No. 7, pp. 1687-1699.
- Khomenko, A., Kharchenko, D. and Yushchenko, O. (2004), "Jamming transition with fluctuations of characteristic acceleration/braking time", *Physics - Statistical Mechanics*, Vol. 37, pp. 44-56.
- Kimiaefar, A., Saidi, A.R., Bagheri, G.H., Rahimpour, M. and Domairry, D.G. (2009), "Analytical solution for van der pol-duffing oscillators", *Chaos, Solitons & Fractals*, Vol. 42 No. 5, pp. 2660-2666.
- Li, Y., Zhu, H., Cen, M., Li, Y., Li, R. and Sun, D. (2013), "On the stability analysis of microscopic traffic car-following model: a case study", *Nonlinear Dynamics*, Vol. 74 Nos 1/2, pp. 335-343.
- Li, Y., Zhang, L., Zheng, H., He, X., Peeta, S., Zheng, T. and Li, Y. (2017), "Nonlane-discipline-based car-following model for electric vehicles in transportation-cyber-physical systems", doi: [10.1109/TITS.2017.2691472](https://doi.org/10.1109/TITS.2017.2691472).
- Lighthill, M.J. and Whitham, G.B. (1955), "On kinematic waves. II: a theory of traffic flow on long crowded roads", *Proceedings of the Royal Society of London A*, Vol. 229 No. 1178, pp. 317-345.
- Lu, C.C., Zhou, X. and Zhang, K. (2013), "Dynamic origin-destination demand flow estimation under congested traffic conditions", *Transportation Research Part C: Emerging Technologies*, Vol. 34, pp. 16-37.
- Maerivoet, S. and De Moor, B. (2005), "Cellular automata models of road traffic", *Physics Reports*, Vol. 419 No. 1, pp. 1-64.
- Momeni, M., Jamshidi, N., Barari, A. and Ganji, D.D. (2010), "Application of he's energy balance method to duffing-harmonic oscillators", *International Journal of Computer Mathematics*, Vol. 88 No. 1, pp. 135-144.
- Nagatani, T. (1998), "Thermodynamic theory for the jamming transition in traffic flow", *Physical Review E*, Vol. 58 No. 4, pp. 4271-4276.
- Nagatani, T. (2000), "Traffic jams induced by fluctuation of a leading car", *Physical Review, E, Statistical Physics, Plasmas, Fluids and Related Interdisciplinary Topics*, Vol. 61 No. 4, pp. 3534-3540.
- Nagatani, T. (2002), "The physics of traffic jams", *Reports on Progress in Physics*, Vol. 65 No. 9, pp. 1331-1386.
- Nagel, K. (1994), "Life times of simulated traffic jams", *International Journal of Modern Physics C*, Vol. 5 No. 3, pp. 567-580.
- Olemskoi, A.I. and Khomenko, A.V. (2001), "Synergetic theory for jamming transition in traffic flow", *Physical Review, E, Statistical, Nonlinear, and Soft Matter Physics*, Vol. 63 No. 3, p. 036116.
- Payne, H.J. (1979), "FREEFLO: a macroscopic simulation model of freeway traffic", *Transportation Research Record*, Vol. 722, pp. 68-77.
- Peng, G., Nie, F., Cao, B. and Liu, C. (2012), "A driver's memory lattice model of traffic flow and its numerical simulation", *Nonlinear Dynamics*, Vol. 67 No. 3, pp. 1811-1815.
- Peng, G.H., Cai, X.H., Cao, B.F. and Liu, C.Q. (2011), "Non-lane-based lattice hydrodynamic model of traffic flow considering the lateral effects of the lane width", *Physics Letters A*, Vol. 375 Nos 30/31, pp. 2823-2827.

- Pourdavish, A. (2006), "A reliable symbolic implementation of algorithm for calculating Adomian polynomials", *Applied Mathematics and Computation*, Vol. 172 No. 1, pp. 545-550.
- Puppo, G., Semplice, M., Tosin, A. and Visconti, G. (2017), "Analysis of a multi-population kinetic model for traffic flow", *Communications in Mathematical Sciences*, Vol. 15 No. 2, pp. 379-412.
- Qian, Y.S., Feng, X. and Zeng, J.W. (2017b), "A cellular automata traffic flow model for three-phase theory", *Physica A: Statistical Mechanics and Its Applications*, Vol. 479, pp. 509-526.
- Qian, Y.S., Zeng, J.W., Wang, N., Zhang, J.L. and Wang, B.B. (2017a), "A traffic flow model considering influence of car-following and its echo characteristics", *Nonlinear Dynamics*, Vol. 89 No. 2, pp. 1099-1109.
- Qu, X., Wang, S. and Zhang, J. (2015), "On the fundamental diagram for freeway traffic: a novel calibration approach for single-regime models", *Transportation Research Part B: Methodological*, Vol. 73, pp. 91-102.
- Richards, P.I. (1956), "Shock waves on the highway", *Operations Research*, Vol. 4 No. 1, pp. 42-51.
- Sánchez-Medina, J.J., Galán-Moreno, M.J. and Rubio-Royo, E. (2010), "Traffic signal optimization in "la almazara" district in Saragossa under congestion conditions, using genetic algorithms, traffic microsimulation, and cluster computing", *IEEE Transactions on Intelligent Transportation Systems*, Vol. 11 No. 1, pp. 132-141.
- Seyed Alizadeh, S.R., Domairry, G.G. and Karimpour, S. (2008), "An approximation of the analytical solution of the linear and nonlinear integro-differential equations by homotopy perturbation method", *Acta Applicandae Mathematicae*, Vol. 104 No. 3, pp. 355-366.
- Tang, T.Q., Huang, H.J., Zhao, S.G. and Shang, H.Y. (2009), "A new dynamic model for heterogeneous traffic flow", *Physics Letters A*, Vol. 373 No. 29, pp. 2461-2466.
- Xiao, W., Chen, Y.G., Yang, Y.P. and Yang, C. (2017), "The impact of intelligent vehicle on the two-route system with a work zone", doi: [10.1142/S0129183117501066](https://doi.org/10.1142/S0129183117501066).
- Yang, S., Deng, C., Tang, T. and Qian, Y. (2013), "Electric vehicle's energy consumption of car-following models", *Nonlinear Dynamics*, Vol. 71 Nos 1/2, pp. 323-329.
- Zhang, X., Onieva, E., Perallos, A., Osaba, E. and Lee, V.C. (2014), "Hierarchical fuzzy rule-based system optimized with genetic algorithms for short term traffic congestion prediction", *Transportation Research Part C: Emerging Technologies*, Vol. 43, pp. 127-142.
- Zhu, Y., Chang, Q. and Wu, S. (2005), "A new algorithm for calculating adomian polynomials", *Applied Mathematics and Computation*, Vol. 169 No. 1, pp. 402-416.

Corresponding authorSafa Bozkurt Coşkun can be contacted at: sbcoskun@yahoo.com