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FLOW-BASED P-HUB MEDIAN INTERDICTION PROBLEM

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FLOW-BASED P-HUB MEDIAN INTERDICTION PROBLEM

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By

Abdulkerim BENLİ

January 2017

FLOW-BASED P-HUB MEDIAN INTERDICTION PROBLEM

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By
Abdulkerim BENLİ
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ACCEPTANCE AND APPROVAL

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ABSTRACT

FLOW-BASED P-HUB MEDIAN INTERDICTION PROBLEM

Abdulkerim BENLİ

MSc. in Industrial Engineering

Supervisor: Assoc. Prof. Dr. İbrahim AKGÜN

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There are two players in a network interdiction problem: a network user who wishes to operate a system optimally, and an opponent/interdictor who tries to prevent the system from operating optimally. Interdiction problems can be modeled as a bi-level min-max or max-min problem in the Stackelberg Game logic. In this thesis, we handle the interdiction problem within the context of the p-hub median problem. The network user solves the problem of locating *p* hubs to minimize the cost associated with operating the network. In response to the network user, the interdictor tries to maximize network user's cost by removing hub characteristics of effective hubs with its limited resources. The p-hub median problem of the network user is modeled on the flow-based networks. The model we develop in this study, unlike the previous literature, does not require the complete network and enables one to find the correct solution in cases that do not provide triangle inequality between nodes. Therefore, this new model provides significant advantages regarding the solution times and modeling capabilities compared to the facility interdiction models offered by the literature.

Keywords: p-hub median interdiction problem, network interdiction, Stackelberg Game, hub disruption, bilevel binary program.

ÖZET

AKIŞ TABANLI P-HUB *(ANA DAĞITIM ÜSSÜ)* ORTANCA ENGELLEME PROBLEMİ

Abdulkerim BENLİ

Endüstri Mühendisliği Bölümü Yüksek Lisans

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Serim önleme/kesme problemlerinde, bir serim üzerinde tanımlı bir sistemi optimal şekilde işletmeye çalışan bir serim kullanıcısı ile sistemin optimal çalışmasını engellemeye çalışan bir rakip/saldırgan olmak üzere iki oyuncu vardır. Problem, Stackelberg Oyunu mantığı içerisinde, iki seviyeli minimaks veya maksimin problemi olarak modellenebilir. Bu çalışmada, serim kesme problemi, p-hub ortanca problemi kapsamında ele alınmıştır. Serim kullanıcısının, maliyeti minimize edecek şekilde ana dağıtım üssü yer seçimi problemi çözdüğü; rakibin ise, sınırlı kaynaklar ile ana dağıtım üslerini kullanılamaz hale getirerek minimum maliyeti maksimize etmeye çalıştığı kabul edilmiştir. Serim kullanıcısın p-hub ortanca problemi, gerçek serim yapıları üzerinde ve akış tabanlı olarak modellenmiştir. Geliştirilen model, daha önceki çalışmalardan farklı olarak, tam serim yapısı gerektirmemekte ve üçgen eşitsizliğini sağlamayan durumlarda da doğru çözüm vermektedir. Önerilen modelin, hem çözüm zamanları hem de modelleme yetenekleri açısından literatürdeki tesis yeri seçimi önleme modellerine göre önemli avantajlar sunduğu görülmüştür.

Anahtar kelimeler: p-hub ortanca üzerinde engelleme, serim engelleme/kesme modeli, Stackelberg oyunu, hub (ana dağıtım üssü) işleyişini durdurma.

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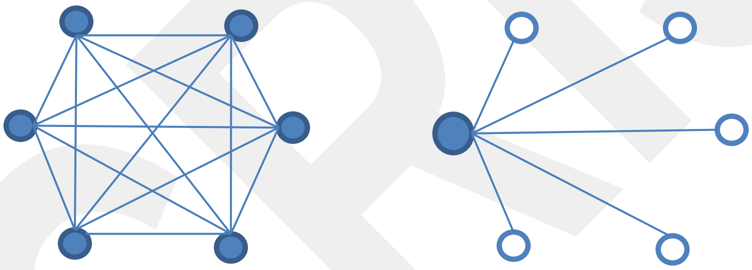
Chapter 1

INTRODUCTION

Point-to-point transportation transfers commodities and passengers directly to their destinations rather than transferring them via central locations. Point-to-point transportation had been the only transport option in package delivery and airline industries until an international package delivery company, FedEx, changed usual transportation rules in the early 1970's. The company created a new network system to carry parcels between the origin and the destination cities. This new system consolidates incoming packages from the origin cities and sorts them according to their destination areas in central facilities. Then, packages are sent to another central facility or delivered directly to the destination city. These central facilities and their connected destination locations are called as hubs and spokes respectively, and the system is called as the hub-and-spoke network. The success of the application of the system in the delivery industry led hub-and-spoke network approach to spreading over airline companies after the deregulation act in 1978 in USA airline industry. The deregulation allowed passengers to fly through central airports/hubs rather than to reach their destination airport directly. On the other hand, various industries also use hub-and-spoke network models. For example, telecommunication networks distribute data packages via big data centers which are interconnected by fiber optic cables that have high transmission capacity.

The main reason behind the prevalence of hub-and-spoke network approach in transportation and telecommunication network systems is its operational efficiency compared to point-to-point approach. This is due to the fact that a point-to-point network connects all origin and destination pair of nodes (locations) without intermediate transfer points. Consider a network with n nodes. There must be $n(n-1)$ links to connect all the nodes to each other, and therefore $n(n-1)/2$ routes are required to create a point-to-point transport. For example, the point-to-point network design depicted in Figure 1.1(a) requires 30 connections between origin and destination pairs, and 15 routes for 6 nodes. If we set one out of n nodes as hub facility, and connect the other nodes to this hub as spokes, the new hub-and-spoke network design only requires $2(n-1)$ connections to connect all origin and destination pairs. For instance, the 6-nodes network with one hub as in Figure 1.1(b) with 10 connections (instead 30 connections as in Figure 1.1(a)) can create 30 origin and destination pairs.



(a) (b)

Figure 1.1. Point-to-Point Transit and Hub-and-spoke Structure

Hub-and-spoke network systems enable a cost-efficient transportation due to the small number of routes. Also, time-consuming and challenging operations, such as sorting and consolidation, are processed in the hubs rather than in each destination. This network system also generates a carbon efficiency by reducing fuel consumption in transportation. Moreover, a new destination can be attached to the system easily in hub-and-spoke networks rather than connect it to every destination as in point-to-point networks. Furthermore, the consolidated flow of passengers, commodities or packages generates economies of scale between hub facilities because of reduced setup and operating expenses per unit of transported flow.

Since the network operators from different sectors facilitate hub-and-spoke systems because of their various advantages, the attention of many researchers has moved towards the problem of finding optimal ways to design such systems. The design of a hub-and-spoke system requires finding the locations of hubs and allocating non-hub nodes to these optimal hubs. Various studies have developed mathematical models to design different types of hub-and-spoke systems.

Even though hub-and-spoke systems provide many benefits, they may also have drawbacks if not properly managed. Shipment delays may occur because of the time spent at the hubs. For instance, airline passengers may have to wait for hours in the hub airports to transfer. Also, if the service flow through the hubs is not sufficient, economies of scale may not be achieved. Therefore, finding the optimal locations for hub facilities is a critical decision for network operators.

On the other hand, hub-and-spoke systems may be disrupted due to adverse events, e.g., natural or human-made disasters and terrorist attacks. The disruption on any part of a hub-and-spoke network may affect the functioning of the whole system. Especially, hubs are vital because a disturbance on hubs may cause the system to fail if necessary precautions are not taken beforehand, or contingency plans are not ready. If a hub is disrupted for some reason, every origin and destination getting service from the disrupted hub are poorly affected. For example, FedEx, a USA package delivery company, lost $125 million in 2014 due to the operation delays over its hub airports with severe weather conditions [1].

Intentional attackers may also disrupt hub-and-spoke systems. For example, USA economy lost about $10 billion (with indirect costs) due to Ronald Reagan hub airport was closed for one month after 9/11 terrorist attack. Furthermore, US Airways, a major USA airline company, had gone bankruptcy due to the canceled flights over this disrupted hub airport [2].

Although the analysis of disruptions is of great importance, only the recent studies have focused on this issue. In facility location studies, there are dozens of models to optimize the location of facilities in different types of environments and under various considerations. However, the studies on the reaction of facility location and hub location models to disruptive events are very new [3]. Except for a few specific ones, most of the studies in the literature assume that hub-and-spoke network systems run in a perfect environment, and no external or internal threats exist for these systems. However, these assumptions do not hold in the real-world. Like every system, the hub-and-spoke networks are also open to numerous possibilities of disruptions. Therefore, the reliability of hubs against disruptions is a key criterion to design a successful network system.

When disruptions are probabilistic as natural disasters are, they can be modeled using stochastic programs. Their effect can be incorporated into the models as exogenous factors. On the other hand, in intentional disruptions, there exist intelligent attackers and their goal is to give the extreme damage to the system. In this sense, malicious attacks are the worst-case disruptions for the hub-and-spoke network systems. Intentional network disruption is an interdiction operation of an intentional attacker (or network interdictor). On the other hand, network operators/users have precautions against those attacks to minimize the damage. Network users are the decision makers guarding themselves against deliberate assaults. In this regard, there are two different agents in the network: the network interdictor and the network user.

These decision makers compete on the same network system. The network user facilitates a hub-and-spoke approach to finding an efficient way to transport services such as commodities, information, and passengers. The network interdictor attacks the network to interrupt the activities of the network user. In other words, these decision makers play a game on the system. Herein, the game refers to following moves of decision makers.

In this thesis, we introduce a two-stage hub interdiction game between two-player; network user and the network interdictor. In this network interdiction game, following moves of the decision makers can be defined as a Stackelberg game in which the leader/intentional attacker moves first, and the follower/ network user acts against that move [4], [5]. The network interdictor firstly attacks the critical facilities of the network user. This is the worst-case scenario for the network user. Therefore he creates a contingency plan to avoid the assault and locate the hub facilities in alternative locations and connects spokes to them in an efficient way. The network user wishes to design a hub-and-spoke system that will be operational even after the worst-case event occurs.

Since game theoretical mathematical models consist of reciprocal actions of decision makers, they are modeled in multi-level mathematical formulations. In this thesis, we formulate the hub-and-spoke network interdiction game with a bi-level program consisting of lower and upper levels. The lower problem of the model gives the network user's problem which minimizes total transportation costs by installing hub facilities. This is simply a p-hub median problem. In the upper level, the network interdictor attempts to worsen the objective value of the network user by interdicting its facilities. Interdiction operations are done by putting penalty costs on critical hubs of the network user's facilities. The penalty costs are sufficient incentives for a network user not to choose the interdicted facilities so that the network user is forced to choose alternative facilities as hub points. Our flow-based p-hub median interdiction problem is a max-min program in which upper and lower levels have maximization and minimization objectives, respectively. We finally apply an algorithm based on Benders decomposition algorithm to solve the model.

Unlike the previous studies on the hub-and-spoke network interdiction, our model allows the network user to construct as many of hub facilities as he wants. For instance, assume that a network user wants to locate p hub facilities on a network and a network interdictor has just enough resources to disrupt r out of them. Previous studies solve this hub interdiction problem for p-r hub facilities. This is the worst-case scenario for the network user who operates a present hub network. This is due to the fact that the network user has to sustain its operation with remaining hub facilities after the interdiction. However, if network system is not present and network user designs a new hub network, this scenario cannot be applicable.

This study considers the hub interdiction problem for the case in which a network user wishes to locate *p* hub facilities with the information of a presence of a network interdictor. The network interdictor finds out the most critical *r* facilities to attack. The network user generates an alternative hub location plan including *p* hubs again regardless interdicted facilities. Thus, the network user can mitigate the possible risks resulted from intentional attacks in initial network design.

Moreover, the model can also be applied to the present hub-and-spoke network systems. In this case, the network user finds out the most critical hub facilities to be attacked by an interdictor and create a contingency plan in the case of a deliberate assault. This emergency plan shows the network user the facilities which are worth to move instead of interdicted facilities.

Furthermore, another advantage of our model is that it can run on more realistic cases than literature due to the fact that the lower level structure of the model allows flexing some assumptions of general hub location problem studies. Further details on the issue can be found in Chapter 2.

The remainder of this thesis is organized as follows: Chapter 2 introduces the hub location problems and the p-Hub Median problem that we consider. Chapter 3 discusses the general network interdiction and facility interdiction problems. Chapter 4 presents our flow-based p-hub median interdiction problem while the computations and the results of the tests run on the model are shown in Chapter 5, and finally, Chapter 6 concludes the thesis and discusses the future implications.

Chapter 2

HUB LOCATION PROBLEM (HLP)

2.1. Background

In real-life networking systems, hub location is a long-term strategic decision. Replacing existing hub facilities with the new ones is both time consuming and costly due to the large setup and initial operation expenses [6]. Therefore, the optimal location of hubs in network design has a great importance for a network user. In general, *Hub Location Problems (HLPs)* attempt to find efficient locations for hub facilities and design the whole hub-and-spoke network to minimize a cost based objective.

However, this is not an easy task. The difficulty with HLP is that there are two different assignments to be made. First, hubs are located in the network, and then non-hub nodes are assigned to these hubs with their flows. Nonetheless, researchers take that arduous task and analyze HLP considering various characteristics and the environment of network systems.

HLPs are broadly applied in transportation and telecommunication networks. The application of HLP in transportation networks includes package delivery, surface transportation, air freight, passenger travel, and urban transportation systems. These networks use hub facilities as transshipment terminals for different modes of vehicles carrying flows from origins. Hubs consolidate flows coming from origins and sort them regarding their destinations. The consolidation functionality generates economies of scale on transportation costs of hub-to-hub connections (also origin-to-hub and hub-to-destination links in some HLPs) [7].

In telecommunication systems, HLPs apply distributed data networks. These networks route electronic data over physical cables such as fiber optic and coaxial cables, or through the air with satellite channels and microwave links. Hubs are hardware equipment such as switches, concentrators, multiplexors, and routers. Economies of scale in data transmission and centralized network architecture motivate hub-and-spoke system to be used in telecommunication [7].

O'Kelly (1986a) was the first study [8] modeled HLP with a continuous mathematical formulation in 1986. Since then, researchers from different disciplines study the HLPs in the areas of location science, telecommunication and computer networks, network optimization, transportation, and geography. The study area expands with the work of [9],[10] for discrete models and Campbell (1994 and 1996)'s new problem definitions [11],[12] on the same objective structure of previous studies. Also, [13],[14] reduce the solution complexity for hub problems with their flow-based approach. Comprehensive survey and review studies on hub networks begin with [15]. Alumur and Kara (2008) classify discrete hub location models [16] until 2008. Campbell and O'Kelly (2012) start to survey with the first hub studies [17] and review recent models and perspectives. Recently, Farahani et al. (2013) review HLPs considering solution methods [18].

2.2. Characteristics of HLP

2.2.1. Properties

Broadly speaking, in facility location problems, service demand of a demand point is supplied from the nearest facility. There are no extra transfer facilities to route flow coming through a demand point or leaving a supply point. However, the most important characteristic of HLPs is to use transshipment points between (O/D) pairs. *Service demand of a demand point (destination) from a supply point (origin) is carried out over transshipment facilities (hubs).* Hub facilities lie in the O/D path. Origins and destinations are interchangeable, which means that a supply point can be a demand point and vice versa. Also, hub facilities can have their service supply and demand. Setup and operational transportation expenses per unit of flow decrease because *economies of scale* on costs of moving consolidated flows between hubs.

Another property of HLPs is that (O/D) pairs can be assigned only to single or multiple hub facilities. *Single allocation HLPs* assign particular hub facilities to each origin and destination point. However, *multiple allocation HLPs* allow (O/D) pairs getting service from various hub facilities. This feature distinguishes HLP from other facility location problems whose demand points can be served only by one supply facility.

Consider hub-and-spoke networks in Figure 2.2.1 which depict different network designs where *i* and *j* nodes represent the *O/D* pairs, and *k* and *m* nodes represent hub facilities in the path of pairs. *i* and *j* are interchangeable supply and demand points, respectively, and hub facilities also can be the origin and the destination points. Figure 2.2.1(a) shows a sample of *single allocation HLP* where *i* and *j* are served by only hub nodes, *k* and *m*, respectively. Figure 2.2.1(b) illustrates a sample *multiple allocation HLP* in which both *k* and *m* hub facilities are assigned to*(i/j)* pair.

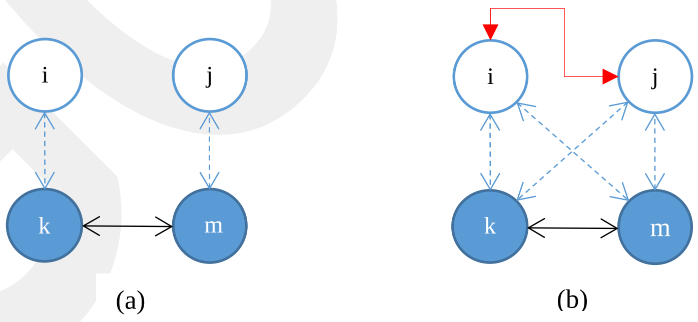


Figure 2.2.1. Hub Location Problems on representative networks

Dashed arrows between non-hub nodes *i*, *j*, and hub nodes *k*, *m* are called *access arcs* and bold straight arrows between hub nodes *k* and *m* are called as *hub arcs* in Figure 2.2.1(a) and 2.2.1(b). Red elbow arrow is a *direct arc* between non-hub nodes *i* and *j* in Figure 2.2.1(c).

2.2.2. Definition of HLP

We can define a general HLP as follows: Consider a complete graph *G= (N, A)* where *N* is the set of origin and destination nodes, and *A* is the set of arcs between these nodes. Let the members of *N* be also potential hub facilities. For each O/D pair *(i, j)*, let W_{ij} denote the amount of the flow and d_{ij} gives the distance from origin $i \in N$ to destination $j \in N$, respectively. A *hub arc* $a = (i,j) \in A$ connects the hub nodes (*i* and *j)* where transportation cost of one unit of flow is $\alpha l_{ij} W_{ij}, for\ 0 < \alpha < 1$. α is the discount factor for the consolidated flow between inter-hub transportation costs. HLPs achieve the economies of scale on transportation cost with this discount factor. On the other hand, for an *access arc* $a = (i,j) \in A$, flow cost is calculated as $d_{ij} W_{ij}$. In this given network structure, *a generic HLP searches the optimal hub facilities among* $n \in N$ *and sets up hub arcs between them, then finally, transfers flows from origin to destination routing via these hubs with the objective of minimizing the setup and flow costs*.

2.2.3. Assumptions

Researchers make six assumptions for HLPs:

1. The underlying network is complete.

2. The economies of scale on transportation costs over the hub-and-spoke network are achieved by the discount factor, α.

3. The direct flows from origins to destinations are not allowed.

4. The Access and Hub arcs have no setup costs.

5. The discount factor is independent of the amount of flow and same for all hub arcs.

6. Distances, d_{ij}, satisfy the triangle inequality.

Assumption 1 is a handicap for HLPs to work on more realistic problems because mostly, real-world networks are not complete. Assumption 3 guarantees that (O/D) pairs must include at least one hub in their paths. Once hubs are located, they are fully interconnected because there is no additional setup cost for hub arcs by assumptions 4 and 6. When assumption 6 is not satisfied O/D paths can include more than two hubs, otherwise each O/D flow is routed over two hubs at most.

Combining all these assumptions; each O/D path has a collection leg from the origin to the first hub, a transfer leg between the first and the last hub, and a distribution leg from the last hub to the destination provided by the third and the fourth assumptions. The flow of O/D pair can be transferred by one hub with no hub arcs or two hubs with one hub arc. Consequently, the flow cost of one O/D pair rises by a basic formula, $F_{ijkm} = d_{ik}W_{ik} + \alpha(d_{km}W_{km}) + d_{jm}W_{jm}$. The first, the second, and the third terms of the formula are collection, transfer and distribution costs along the O/D path, respectively. The objective of HLP is generally the summation of this formula over all O/D pairs [7].

2.3. Classification

Depending on characteristics of HLPs such as network topology, objective, and decision maker preference, HLP is classified into several categories in the literature. One classification is the objective of the problems:

2.3.1. Objectives

Broadly speaking, the objective of HLPs is minimizing the transportation costs or travel times in transportation applications. In telecommunication networks, HLPs focus on setup costs over the hub-and-spoke architecture [7]. Analogous to facility location problems such as *p-median, p-center and covering problems*, HLPs can be classified as follows.

p-Hub Median Problems consider of locating *(p-)* number of hub facilities to minimize total flow cost overall O/D paths on hub-and-spoke networks.

p-Hub Center Problems minimize the maximum flow cost of all the O/D pairs, all the arcs of the network or access arcs.

Hub Covering Problems minimize the hub setup costs to achieve at least a specified service level (travel distance). Service demand is satisfied only its origin and destination are in the range of a hub facility.

2.3.2. Allocation Type and Capacity Limitation

We can also classify HLPs according to the hub allocation type of O/D pairs as single and multiple allocations. In *single allocation* HLPs, one hub facility is assigned to each origin and destination. *Multiple allocation* HLPs allow multiple hub facilities to be assigned to origins and destinations.

Moreover, the capacity limitation is another type of categorization for HLPs. *Uncapacitated* hub problems put no capacity limit to arcs of hub networks. *Capacitated* ones may restrict flow capacity of the hub and access arcs.

2.4. Mathematical Models

This thesis considers a flow-based p-hub median interdiction problem. Therefore, we will only focus on *p-hub median hub location problems* (pHLPs). For a broad review of hub models having different objectives, see [16], [18].

2.4.1. Single Allocation p-Hub Median Problem

O’Kelly (1987) was the first study [10] developed a quadratic formulation for *single allocation uncapacitated p-hub location model* (USApHMP) as follows:

Indices and Sets:

N is the set of nodes in the hub-and-spoke network.

$i \in N$ represents an origin node.

$j \in N$ represents a destination node.

$k \in N$ represents the hub node in the collection leg of a O/D path.

$m \in N$ represents the hub node in the distribution leg of a O/D path.

Data:

p denotes the number of hub facilities to be located.

α is the discount factor on hub arcs.

d_{ij} is the distance between *node i* and *node j*.

Decision Variables:

W_{ij} is the amount of flow between *node i* and *node j*.

Z_{ik} is equal to *1* if *node i* is allocated by *hub node k*, *0* otherwise.

Z_{kk} is equal to *1* if *node k* is a hub, *0* otherwise.

$O_i = \sum_j W_{ij}$ total amount of flow going from node *i*.

$D_i = \sum_j W_{ji}$ total amount of flow coming from node *i*.

USApHMP formulation [10]**:**

$$\min \sum_i \sum_k (O_i + D_i) Z_{ik} d_{ik} + \sum_i \sum_j \sum_k \sum_m \alpha d_{km} W_{ij} Z_{ik} Z_{jm} \tag{2.4.1}$$

$$s.t. \qquad Z_{ik} \le Z_{kk} \qquad i,k \in N \tag{2.4.2}$$

$$\sum_k Z_{ik} = 1 \qquad i \in N \tag{2.4.3}$$

$$\sum_k Z_{kk} = p \tag{2.4.4}$$

$$Z_{ik} \in \{0,1\}; W_{ij} \ge 0 \qquad i,j,k \in N \tag{2.4.5}$$

where $O_i = \sum_j W_{ij}$ and $D_i = \sum_j W_{ji}$. Objective (2.4.1) minimizes the total transportation cost over the network with reduced cost of hub flows by using $0 < \alpha < 1$. The objective has quadratic terms due to the multiplication of integer hub variables, Z. Constraints (2.4.2) enforce flows, once they are located, to route via hubs. (2.4.3) ensure that each node is assigned to a single hub, and (2.4.4) locates the p-hubs in the network.

A different approach to p-hub median problems is tracing flows on each O/D path. Campbell (1996)'s *path-based USApHMP* [12] introduces a binary decision variable; X_{ijkm} is equal to *1* if the flow from *origin i* to *destination j* goes through *hubs k* and *m*, *0* otherwise. Due to four indices of the decision variable, the model leads to very large formulations to deal with. Therefore, Skorin-Kapov et al. (1996) relaxed the binary X_{ijkm} and proposed a new mixed integer program [19].

Indices and Sets:

N is the set of nodes in the hub-and-spoke network

$i \in N$ represents an origin node.

$j \in N$ represents a destination node.

$k \in N$ represents the hub node in the collection leg of a O/D path.

$m \in N$ represents the hub node in the distribution leg of a O/D path.

Data:

α is the discount factor on hub arcs.

d_{ij} is the distance between *node i* and *node j*.

Decision Variables:

W_{ij} is the amount of flow between *node i* and *node j*.

$F_{ijkm} = d_{ik}W_{ik} + \alpha(d_{km}W_{km}) + d_{jm}W_{jm}$ flow cost over a O/D path.

Z_{ik} is equal to *1* if *node i* is allocated by *hub node k, 0* otherwise.

X_{ijkm} is demand service level from *origin i* to *destination j* goes through *hubs k* and *m,* and takes value between *0* and *1*.

USApHMP Relaxation Formulation [19]**:**

$$\min \sum_i \sum_j \sum_k \sum_m F_{ijkm} X_{ijkm} \qquad (2.4.6)$$

$$s.t. \qquad (2.4.2) – (2.4.5)$$

$$\sum_m X_{ijkm} = Z_{ik} \qquad i,j,k \in N \qquad (2.4.7)$$

$$\sum_k X_{ijkm} = Z_{jm} \qquad i,j,m \in N \qquad (2.4.8)$$

$$X_{ijkm} \geq 0 \qquad i,j,k,m \in N \qquad (2.4.9)$$

where $F_{ijkm} = d_{ik}W_{ik} + \alpha(d_{km}W_{km}) + d_{jm}W_{jm}$. Constraints (2.4.7) require that if a collected flow from node *i* reaches hub *k*, then it should be transferred to another hub *m* and (2.4.8) ensure that a distribution from hub *m* to destination *j* be made if and only if it comes from another hub *k*. The key feature of the model is providing tight linear relaxation bounds although having $O(n^4)$ decision variables $O(n^3)$ constraints.

Flow-based formulations are also developed for USApHMP defining flows on hub arcs as multicommodity flows where each commodity represents origin's identity. First flow-based model is proposed by Ernst and Krishnamoorthy (1996) and it is as follows [13]:

Indices and Sets:

N is the set of nodes in the hub-and-spoke network
$i \in N$ represents an origin node.
$j \in N$ represents a destination node.
$k \in N$ represents the hub node in the collection leg of a O/D path.
$m \in N$ represents the hub node in the distribution leg of a O/D path.

Data:

α is the discount factor on hub arcs.

d_{ij} is the distance between *node i* and *node j*.

Decision Variables:

W_{ij} is the amount of flow between *node i* and *node j*.

Z_{ik} is equal to *1* if *node i* is allocated by *hub node k*, *0* otherwise.

Y^i_{km} as the total amount of flow originating from node *i* and travelling through hubs *k* and *m*.

$O_i = \sum_j W_{ij}$ be the total amount of flow originating from node i.

$D_i = \sum_j W_{ji}$ be the total amount of flow sinking at node *i*.

Flow-based USApHMP Formulation [13]*:*

$$\min \sum_i \sum_k (O_i + D_i) d_{ik} Z_{ik} + \sum_i \sum_k \sum_m \alpha d_{km} Y^i_{km} \qquad (2.4.10)$$

$$s.t. \qquad (2.4.2) – (2.4.5)$$

$$\sum_j W_{ij} Z_{jk} + \sum_m Y^i_{km} = \sum_m Y^i_{mk} + O_i Z_{ik} \qquad i,k \in N \qquad (2.4.11)$$

$$Y^i_{km} \geq 0 \qquad i,k,m \in N \qquad (2.4.12)$$

Constraints (2.4.11) are the flow balance constraints for commodity *i* at node *k* where the allocation pattern determines the demand and supply at each node. This formulation provides tracking every flow regarding the origins rather than each origin-destination pair. Instead of four-subscripted approach in the previous models, three-subscripted variables are used. Their formulations require $O(n^3)$ decision variables. The drawback of the formulations is weak LP relaxations. However, large scale problems *(in number of nodes)* can be solved with the model because of the simplicity of the solution approach and computational advantage with fewer variables [20]. Furthermore, Contreras et al. (2010) derived a family of valid inequalities which can be used to tighten formulation of the model [21].

2.4.2. Multiple Allocation p-Hub Median Problem

Multiple allocation pHLP allows nodes to be allocated to multiple hubs to satisfy their demand and exhaust their supply. Campbell (1992) introduces the first *Uncapacitated Multiple Allocation p-Hub Median Problem* (UMApHMP) model [22].

Indices and Sets:

N is the set of nodes in the hub-and-spoke network.

$i \in N$ represents an origin node.

$j \in N$ represents a destination node.

$k \in N$ represents the hub node in the collection leg of a O/D path.

$m \in N$ represents the hub node in the distribution leg of a O/D path.

Decision Variables

Z_{kk} is equal to *1* if *node k* is a hub, *0* otherwise.

X_{ijkm} is demand service level from *origin i* to *destination j* goes through *hubs k* and *m,* and takes value between *0* and *1*.

UMApHMP Formulation [22]**:**

min (2.4.6)

$s.t.$ (2.4.4), (2.4.5) and (2.4.9)

$$\sum_{k}\sum_{m} X_{ijkm} = 1 \qquad (2.4.13)$$

$$X_{ijkm} \leq Z_{kk} \quad i,j,k,m \in N \qquad (2.4.14)$$

$$X_{ijkm} \leq Z_{mm} \quad i,j,k,m \in N \qquad (2.4.15)$$

Constraints (2.4.13) ensure that the flow of every (O/D) pair is routed over the hub facilities. Constraints (2.4.14 and 2.4.15) allow only the flows go through *node k* and *node m* if they are chosen as hubs. Even though the decision variable, *X,* is continuous, it only takes the value of *1* and *0,* so that provides tight bounds to the objective. This model does not restrict origins and destinations to be allocated by a single hub.

Moreover, Ernst and Krishnamoorthy (1998) [14] introduce *flow-based Uncapacitated Multiple Allocation p-Hub Median Problem* as follows:

Indices and Sets:

N is the set of nodes in the hub-and-spoke network.

$i \in N$ represents an origin node.

$j \in N$ represents a destination node.

$k \in N$ represents the hub node in the collection leg of a O/D path.

$m \in N$ represents the hub node in the distribution leg of a O/D path.

Data:

α is the discount factor on hub arcs.

d_{ij} is the distance between *node i* and *node j*.

Decision Variables:

$O_i = \sum_j W_{ij}$ be the total amount of flow originating from node i.

$D_i = \sum_j W_{ji}$ be the total amount of flow sinking at node *i*.

W_{ij} is the amount of flow between *node i* and *node j*.

Z_{ik} is the amount of flow *i* to hub *k*.

Y_{km}^i as the total amount of flow originating from node *i* and travelling through hubs *k* and *m*.

H_k is *1* if node *k* is a hub, 0 otherwise.

X_{ijm} be the amount of flow originating from *i* and sinking at *j* going through hub *m*.

Flow-based UMApHMP Formulation [14]**:**

$$\min \sum_i \sum_k d_{ik} Z_{ik} + \sum_i \sum_k \sum_m \alpha d_{km} Y_{km}^i + \sum_i \sum_j \sum_m d_{jm} X_{ijm} \qquad (2.4.16)$$

$$s.t. \qquad \sum_k H_k = p \qquad (2.4.17)$$

$$\sum_k Z_{ik} = O_i \qquad i \in N \qquad (2.4.18)$$

$$\sum_{m} X_{ijm} = W_{ij} \qquad i,j \in N \qquad (2.4.19)$$

$$Z_{ik} + \sum_{m} Y_{km}^{i} = \sum_{m} Y_{mk}^{i} + \sum_{j} X_{ijm} \qquad i,k \in N \qquad (2.4.20)$$

$$Z_{ik} \leq O_i H_k \qquad i,k \in N \qquad (2.4.21)$$

$$\sum_{j} X_{ijm} \leq D_j H_m \qquad i,j,m \in N \qquad (2.4.22)$$

$$Z_{ik}, X_{ijm}, Y_{km}^{i} \geq 0 \qquad i,j,k,m \in N \qquad (2.4.23)$$

Constraints (2.4.18) ensure that all flows from origin *i* is sent to a subset of hubs. (2.4.19) satisfy the demand of destination nodes. (2.4.20) is the well-known flow conservation constraint. Constraints (2.4.21) and (2.4.22) force (O/D) pairs to include at least one hub in their path. The formulation has $O(n^3)$ variables and $O(n^2)$ linear constraints however weak bounds. Marin et al. tighten bounds and improve a new formulation [20], as follows, for the cases which do not satisfy triangle inequality:

Indices and Sets:

N is the set of nodes in the hub-and-spoke network.

$i \in N$ represents an origin node.

$j \in N$ represents a destination node.

$k \in N$ represents the hub node in the collection leg of a O/D path.

$m \in N$ represents the hub node in the distribution leg of a O/D path.

Decision Variables:

W_{ij} is the amount of flow between *node i* and *node j*.

Z_{ik} is the amount of flow *i* to hub *k*.

Y_{km}^{i} as the total amount of flow originating from node *i* and travelling through hubs *k* and *m*.

H_k is *1* if node *k* is a hub, 0 otherwise.

X_{ijm} be the amount of flow originating from *i* and sinking at *j* going through hub *m.*

$O_i = \sum_j W_{ij}$ be the total amount of flow originating from node i.

Flow-based UMApHMP Tightened Formulation [20]:

$$\min \quad (2.4.16)$$

$$s.t. \quad (2.4.19) - (2.4.23)$$

$$\sum_m Y_{km}^i = O_i H_k \quad i,k \in N \quad (2.4.24)$$

$$\sum_k Y_{km}^i = O_i H_m \quad i,m \in N \quad (2.4.25)$$

$$\sum_m Y_{km}^i = Z_{ik} \quad i,k \in N \quad (2.4.26)$$

$$Z_{ii} \geq O_i H_i \quad i \in N \quad (2.4.27)$$

$$X_{ijj} \geq W_{ij} H_j \quad i,j \in N \quad (2.4.28)$$

Constraints (2.4.24) and (2.4.25) force flows to go from one hub to another. Constraints (2.4.27) and (2.4.28) allow hubs to satisfy their demand and exhaust their supply. Constraints (2.4.26) limit the number of nodes traversed by a flow, using the flow which directly comes from the origin as a bound on the outcoming flow.

2.5. A New Model Development

2.5.1. Generalized Uncapacitated Multiple-Allocation p-Hub Median Problem

Pioneering pHLP models work with assumptions listed in Section 2.2.3. Even these are invalid for most real-world hub networks. Therefore, Akgün et al. (2017) introduce *Generalized Uncapacitated Multiple-Allocation p-Hub Median Problem,* G-MApHMP [23], to add more realism to pHLPs with a new problem setting and modeling approach allow several basic assumptions about PHLPs to be relaxed and provide flexibility in modeling several characteristics of real-life hub networks. G-MApHMP can be used with any network and work correctly whether the costs satisfy the triangle inequality or not. It does not impose any structure, e.g., the number of hubs on any route between an origin-destination pair is not limited, and allow several extensions such as direct arcs between non-hub nodes. G-MApHMP is explained in detail as follows [23].

2.5.1.1. Problem Definition

Consider a surface transportation network $G = (N, E)$ with node set $N = \{1, \dots, n\}$ and edge set E. We assume that each edge is undirected and that the network is connected. A subset S of the node set is distinguished as the demand-generating node set. If node i is in S, then it generates a positive annual flow w_{ij} for at least one node $j \in N - \{i\}$. Let D_i be the set of nodes j for which $w_{ij} \geq 0$. This set is defined and nonempty for each node $i \in S$. Define the demand set D to be the union of D_i, $i \in S$. Note that many nodes can be both in S and D at the same time. The objective is to deliver the flows w_{ij} from nodes in S to nodes in D via hubs whose locations are to be determined.

Potential hubs are determined considering not only the nodes but also the edges of the network. The hub-to-hub portions of journeys from sources to sinks are done by more specific, large vehicles to achieve economies of scale and hence the costs for these parts of the network are discounted. However, for surface transportation networks, all parts of the network may not be appropriate for all types of vehicles. For example, there may be bridges on some roads that restrict the passage of oversize vehicles (e.g., 18-wheelers) or some sea lines of communication may not be appropriate for large vessels. In this regard, the edges of the network are differentiated depending on their suitability for hub-to-hub transportation and for discounting. Let E^* be a subset of the edge set that is distinguished as the set of edges that are suitable for specialized vehicles and hence for discounting. Let N^* be the set of nodes that are incident to at least one edge in E^*. We restrict the set of potential hubs to N^*or to a subset H of N^* if other physical, administrative, or legal considerations lead to the elimination of certain nodes in $N^*$from being hubs. Let $G^* = (N^*, E^*)$ be the subnetwork of G consisting of edges that are available for hub-to-hub transportation. Then, if hub-to-hub movement occurs in $G^* = (N^*, E^*)$, then transportation costs of the edges $\{i, j\} \in E^*$ are discounted. If these edges are used for the transportation between non-hub and hub nodes, then no discounting occurs. It is also possible that hub-to-hub transportation occurs on the edges $\{i, j\} \notin E^*$ if it is cheaper to use those arcs and specialized vehicles can use them. For these edges, it is assumed that no discounting is allowed. (However, if necessary, the adopted modeling approach allows discounting.)

Figure 2.5.1 shows a sample network where dashed lines represent roads in E^* and numbered points represent the nodes in N. The nodes at the borders of the rectangular map can (will) be dropped from further consideration. The set of potential hub nodes $N^* = H$ is $\{1, 2, 3, 20, 21, 11, 12\} \cup \{5, 19, 21, 18, 13\} \cup \{15, 14, 13, 12, 10, 8\}$. The special vehicles are to use the dashed roads.

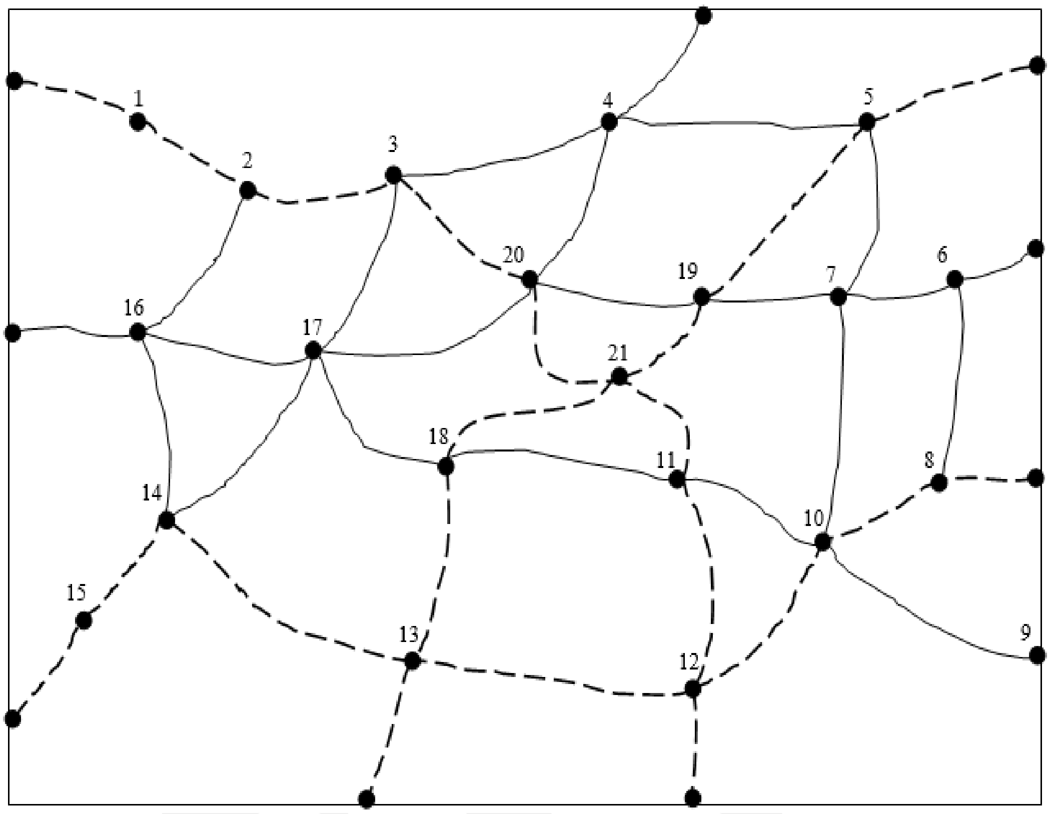


Figure 2.5.1. A network where the dashed lines are highways

Let l_{ij} be the length of edge {i, j} and c_{ij} be the cost of moving one unit of flow per unit length along the edge {i, j}. If the volume of units between nodes *i* and *j* is W_{ij}, the cost of transportation on the edge {i, j} is $c_{ij}\ l_{ij}\ W_{ij}$. For an edge $\{i, j\} \in E^*$, the transportation cost is assumed to be discounted by a factor of α_{ij}. Transportation cost over an edge, $\{i, j\} \in E^*$, is $\alpha_{ij} c_{ij}\ l_{ij}\ W_{ij}$. Total transportation cost on the network is obtained by summing costs over all edges. *The problem is to choose p nodes from a predefined set $H \subseteq N^*$ and find routes from sources in S to demands in D that visit at least one node in the selected hub set so that the total transportation cost is minimized.*

2.5.1.2. Model Development

G-MApHMP is modeled as a multi-commodity flow problem with side constraints. Let $G = (N, A)$ be the directed version of $G = (N, E)$ where each undirected edge $\{i, j\} \in E$ is replaced by a pair of directed arcs (i, j) and (j, i) with each arc having the same length, $l_{ij} = l_{ji}$. We create two copies of $G = (N, A)$, designated as $G_1 = (N_1, A_1)$ and $G_3 = (N_3, A_3)$ with $N_1 = \{11,12, \ldots, 1n\}, N_3 = \{31,32, \ldots, 3n\}, A_1 = \{(1i, 1j) : (i, j) \in A\}$ and $A_3 = \{(3i, 3j) : (i, j) \in A\}$. The lengths of arcs $(1i, 1j)$ and $(3i, 3j)$ are each l_{ij}.

Distinguish $|S|$ commodities each being associated with a supply node $i \in S$. The nodes in $G_1 = (N_1, A_1)$ are supply nodes and the nodes in $G_3 = (N_3, A_3)$ are demand nodes. A node *i* is a supply node *(1i)* in *G1,* and also a demand node *(3i)* in *G3*.

Define $W_i = \sum_{j \in D_i} w_{ij}$ as the total outbound flow at supply node $i \in S$, i.e., $1i \in N_1$. Define also, for each $i \in S$, a supply of W_i units for commodity *i* at node $1i \in N_1$. For all other commodities $i' \in S, i' \neq i$, the supply of commodity i' at node *1i* is zero (not defined). For each demand node $j \in D$, define a demand of w_{ij} units at node $3j \in N_3$ for each commodity *i* for which $j \in D_i$ (i.e., for each $i \in S$ for which $w_{ij} > 0$).

Now two layers of the final network are formed, the first layer being G_1 and the third layer being G_3 with nodes in G_1 taken as sources and nodes in G_3 taken as sinks. The middle layer $G_2 = (N_2, A_2)$ is defined by the edges in E^* that are edges available for hub-to-hub transportation. Define $N_2 = N^*$ and $A_2 = A^*$ where $A^* = \{(i, j) \in A : (i, j) \in E^*\}$. That is, each undirected edge $\{i, j\} \in E^*$ is replaced by a directed pair of arcs (i, j) and (j, i) and such arcs form the new arc set $A^* = A_2$.

The three layers, $G_1, G_2, and\ G_3$ are connected to each other by arcs of the form $(1i, 2i)$ and $(2i, 3i)$ for every $i \in H$. That is, the nodes available for locating $2|H|$ arcs running from the first copy of *i* to the second copy and from the second copy to the third copy. Let $A_{12} = \{(1i, 2i) : i \in H\}$ and $A_{23} = \{(2i, 3i) : i \in H\}$. Finally, the network in the model is $G_0 = (N_0, A_0)$ where $N_0 = N_1 \cup N_2 \cup N_3$, $A_0 = A_1 \cup A_2 \cup A_3 \cup A_{12} \cup A_{23}$.

Figure 2.5.2 shows how the new network is constructed from a 5-node undirected network G. In figure 2.5.1.2, $E^* = \{\{4,5\},\{3,4\},\{2,4\}\}$, $N^* = \{2,3,4,5\}$, and $H = \{3,4,5\}$. Note that G_2 may be disconnected. The graph structure and the flows in the final graph are not sensitive to that.

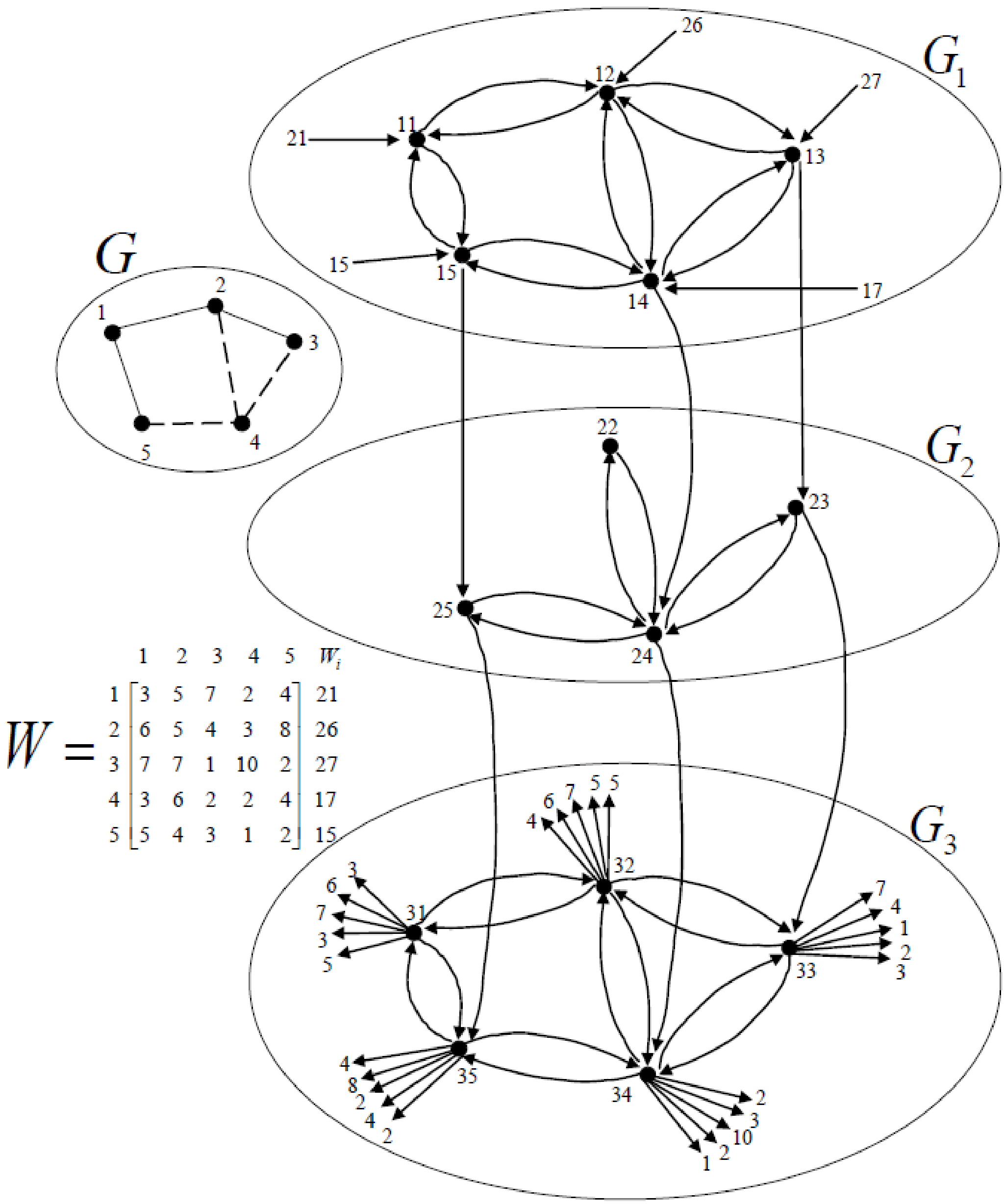


Figure 2.5.2. A schematic representation of the transformed network. Dashed lines in the original network G represent the arcs assumed to be appropriate for hub-to-hub transportation

Define a flow variable X_{ak} for each arc $a \in A_0$ and each commodity $k \in S$. Define a node variable $Y_i \in \{0,1\}$ for each $i \in H$ where $Y_i = 1$ if node i is a hub, 0 otherwise. The flow-based formulation of this problem has a flow conservation equation for each node in N_0 and each commodity k. We choose p hubs via $\sum_{i \in H} Y_i = p$. Controlling the flows in arcs by permitting flows in vertical arcs (those in A_{12} and A_{23}) only if their end nodes are selected as hubs (i.e., arcs *(1i,2i)* and *(2i,3i)* allow the passage of flows only if $Y_i = 1$).

Parameters of the model: l_a, the total cost of moving one unit flow on the arc a, a refers to $i \to j$. l_a takes different values for different types of arcs as in Eq. (2.5.1). In arcs of *G1* and *G3*, l_a is $c_{ij} l_{ij}$. However, in hub arcs of G2, transportation cost is discounted by α_{ij}.

$$l_a = \begin{cases} c_{ij} l_{ij} & if\ a = (1i, 1j) or\ a = (3i, 3j), (i,j) \in A \\ \alpha_{ij} c_{ij} l_{ij} & if\ a = (2i, 2j), (i,j) \in A^* \\ 0 & if\ a = (1i, 2i) or\ a = (2i, 3i), i \in H \end{cases} \quad (2.5.1)$$

Let β be any node of N_0. With abuse of notation, we write $\beta \in S$ if $\beta = 1i$ for some $i \in S$ and $\beta \in D$ if $\beta = 3i$ with $i \in D$. Similarly, we write $\beta \in H$ if $\beta = 2i$ with $i \in H$ and $\beta \in N_2$ if $\beta = 2i$ with $i \in N^*$. The requirement $b_{\beta k}$ at node β for commodity k is defined to be $W_\beta = \sum_{j; w_{ij} \geq 0} w_{ij}$ if $\beta = 1i$ with $i \in S$ and $-w_{k\beta} = -w_{kj}$ if $\beta = 3j$ with $i \in D$. Define $b_{\beta k} = 0$ for all other nodes and $k \in S$.

F_β^{out} is the forward star of a node $\beta \in N_0$ consisting of arcs whose tail is β and F_β^{in} be the inward star of node $\beta \in N_0$ consisting of arcs whose heads are β.

G-MApHMP Formulation, [23]:

$$min\ Z^* = \sum_{k \epsilon S} \sum_{a \epsilon A_0} l_a X_{ak} \tag{2.5.2}$$

$$s.t. \quad \sum_{a \in F_\beta^{out}} X_{ak} - \sum_{a \in F_\beta^{in}} X_{ak} = b_{\beta k} \quad \beta \in (N_1 \cup N_2 \cup N_3), k \in S \tag{2.5.3}$$

$$\sum_{i \in H} Y_i = p \tag{2.5.4}$$

$$X_{(1i,2i)k} \leq W_k Y_i \quad i \in H, k \in S \tag{2.5.5}$$

$$X_{(2i,3i)k} \leq W_k Y_i \quad i \in H, k \in S \tag{2.5.6}$$

$$X_{ak} \geq 0 \quad a \epsilon A_0, k \in S \tag{2.5.7}$$

$$Y_i \epsilon \{0,1\} \quad i \in H \tag{2.5.8}$$

The objective function (2.5.2) minimizes the total transportation cost. Constraints (2.5.3) are the flow conservation constraints for all nodes in the network, (2.5.4) ensure that the number of hubs is *p*. Constraints (2.5.5) ensure that a node in the first layer is connected to its corresponding node in the second layer, if the node in the second layer is chosen as a hub. Constraints (2.5.6) require that a node in the second layer be connected to its corresponding node in the third layer if the node in the second layer is chosen as a hub. This formulation has $O(n^3)$ variables and $O(n^2)$ constraints.

In chapter 3, we discuss network interdiction models in which nodes (e.g. facilities) and arcs (e.g., roads) are assumed to be disrupted. In Chapter 4, we build an p-hub median interdiction model based on G-MApHMP [23].

Chapter 3

NETWORK SYSTEMS UNDER INTENTIONAL ATTACKS

Network systems do not always run in the perfect environment. Disruption events may inflict heavy damages on networks. These disruptions may come from nature itself such as natural disasters and environmental disturbances. For instance, Great East Japan earthquake in March 2011 crippled Japanese economy. Automobile manufacturers could not sustain their supply chains for months, and they lost billions of USD in addition to more direct financial casualties [24]. In the second half of 2011, severe floods caused heavy economic losses in Thailand by USD 40 billion and reduced the country's manufacturing capacity [25]. Unfortunate experiences like these examples increase awareness in the business world. A recent study shows that 80 percent of the firms around the world considers the protection of supply chain networks as a top priority [26].

In addition to natural threats, disruptions may be set up intentionally. For example, a wooden made trestle in Sacramento railroad in the USA is burned down by an arsonist in 2007. Even though the trestle was just 300 feet long, it required rebuilding the structure in a timely fashion. While the trestle was inoperable, trains had to reroute over another railroad. This detour added extra 125 miles on impacted routes [27]. In 2013; crime groups coordinated attack on electricity grid of Mexico's southern state Michoacán that left 420.000 residents without power [28]. USA economy lost 10 billion dollars due to the shutdown of Ronald Raegan hub airport after 9/11 terrorist attacks [2]. Cyber-attacks to

Ukraine’s power grid left more than 200.000 people in darkness for 6 hours in 2015 [29].

A deliberate attack aims maximum damage on a network system. Therefore, interdiction operations are considered as worst-case scenarios. For example, USA electricity grid has 55.000 substations to sustain power in the country. However, interdicting nine out of those stations may cause a complete blackout due to interdependency structure of the grid. An intentional attacker who wishes to damage to USA’s power system most likely strikes these nine critical subsystems. This example is the worst-case scenario for USA electricity system. Broadly speaking out of this example, intentional attackers attempt to interdict critical infrastructure of a network system that is essential to sustain operations of the network. Because when critical infrastructure is under attack, the whole system is in danger.

Determining a risk policy against natural and human-made threats helps to reduce catastrophic consequences of failed systems. Resilient and robust network systems can be achieved by proactive precautions [30]. Ex-ante countermeasures such as contingency plan and risk mitigation strategies are proactive management applications that are based on stochastic approach considering what-can-go-wrong scenarios.

Researchers consider disruption/interdiction of network systems under network interdiction problem. This problem analyses worst-case scenarios that are most pessimistic events may occur in the network.

3.1. Network Interdiction Problem

3.1.1. Background

We interpret network interdiction problem in the context of a two-player game. One player is the network operator/user (NU) who wishes to operate a network with some objectives such as transporting services via the *shortest path* or achieving the *maximum amount of flow* across the network. Another player, called network interdictor (NI), tries to worsen NU's objectives with its capabilities which can change the structure of the network. Disconnecting vertices or edges, delay NU's operations by disrupting facilities, increasing detection probability of NU's activities are a few examples of NI's capabilities. NI has limited resources to implement its plan against NU.

Network interdiction problem is a two-stage game. NI makes the first move by attacking components of the network. NU makes following move on the resulted network after interdiction. Hence, network interdiction problem is a truly Stackelberg game in which the leader/interdictor moves first and the follower/operator acts against that move [4], [5]. If there is no human interdictor, natural threats take the role of NI in worst-case analyses. That is, "Murphy's Law", the pessimistic approach that the worst event will occur, change the network structure [31].

Network Interdiction problems are formulated by usually bi-level structures where each level reflects the problem of the decision maker, e.g., NU and NI. Bi-level models in contrast to single-level models are often integer or mixed integer programs that model the decision-making of players sequentially in the same formulation. The interaction between the interdictor and the defender is modeled simultaneously. The objective function in multi-level models is one that typically reflects pure competition, with the interdictor seeking to minimize an overall network metric (such as flow or satisfied demand) and the follower looking for maximizing this minimum metric. In other words, the follower's job is to minimize the effect of interdiction on their network operations [32].

On a side note, fortification problems are three-stage games with two players, e.g. NU and NI that also take into account of protective actions of NU against NI. In this game, NU acts first and fortify some network components with limited resources anticipation of a possible attack plan of NI. After that, NI assault network to hurt NU's objective. Then, NU put its decision on the resulted network. Fortification problems are modeled with three-level mathematical formulations because of the three-stage game structure.

3.1.2. Generic Interdiction Models

Since the pioneering work [33] of Wollmer (1964) which analyzed the sensitivity of a network when prescribed arcs are removed from a transportation system to minimize the flow, network interdiction problem has been studied in various settings and for network applications such as military, homeland security, and computer networks. However, we will focus on three generic interdiction problems where network user's problem is the *shortest path, maximum flow or minimum cost of flow problems*. For comprehensive literature reviews on different network interdiction problems; see [31], [34].

3.1.2.1. Maximum Flow Interdiction

Wood (1993) defines the *maximum flow network interdiction problem* (MFNIP)[35] in which enemy strikes with limited resources to minimize the maximum flow through a capacitated network and proves that the problem is NP-complete. In the problem, NU attempts to traverse from a source to a sink node while NI tries to disrupt those arcs in the path lies between the source and the sink nodes. NI must expense a necessary resource to break one arc that is limited with an interdiction budget.

Before proceeding with model development of MFNIP, note that we do not give every detail of the problem here. See [35] for further details.

Consider a directed graph $G = (N, A)$ with a node set and an arc set, N and A, respectively. *MFNIP minimizes the maximum flow between source and sink nodes, s and $t \in N$, respectively.* MFNIP is modeled as following bi-level formulation.

Indices:

$i \text{ and } j \in N$ represent nodes of the network G.

$s \text{ and } t \in N$ are source and sink nodes, respectively.

$(i\ ,j)$ represents the arc between nodes *i* and *j*.

Data:

u_{ij} is the capacity level of arc *(i, j).*

u_{ts} is the dummy arc between source and sink nodes, s and t.

Γ is the budget set of the interdictor.

r_{ij} is the amount of resource to interdict arc *(i, j)*

R is the total interdiction budget

NI's Decision Variable:

$\gamma_{ij} \in \Gamma$ is the NI's binary decision variable. γ_{ij} is equal to 1, if arc (i, j) is interdicted, otherwise it is 0.

NU's Decision Variable:

x_{ij} is the NU's decision variable and shows the amount of flow to be routed source node s to sink node t.

MFNIP Bi-level Formulation [35]**:**

$$\min_{\gamma_{ij} \in \Gamma} \max_{x} x_{ts} \qquad (3.1.2.1)$$

$$s.t. \quad \sum_j x_{sj} - \sum_j x_{js} - \sum_j x_{ts} = 0, \qquad (3.1.2.2)$$

$$\sum_j x_{ij} - \sum_j x_{ji} = 0, \qquad i \in N \backslash \{s,t\} \qquad (3.1.2.3)$$

$$\sum_j x_{tj} - \sum_j x_{jt} + x_{ts} = 0, \qquad (3.1.2.4)$$

$$x_{ij} - u_{ij}\left(1 - \gamma_{ij}\right) \leq 0, \qquad (i,j) \in A \qquad (3.1.2.5)$$

$$x_{ij} \geq 0, \qquad (i,j) \in A \cup \{(t,s)\} \qquad (3.1.2.6)$$

$$where\ \Gamma \equiv \left\{\gamma_{ij} | \gamma_{ij} \in \{0,1\}, \forall (i,j) \in A, \sum\nolimits_{(i,j)\in A} r_{ij}\gamma_{ij} \le R\right\}$$

Objective (3.1.2.1) minimizes the maximum flow. The inner maximization problem is a capacitated maximum flow problem with usual flow conservation constraints (3.1.2.2) - (3.1.2.4). Constraints (3.1.2.5) are arc capacity constraints under the control of interdictor's variable on the right-hand side. Interdicting an arc *(i, j)*, is simply reducing its capacity, u_{ij}, to zero. Interdictor spends r_{ij} of resources to make this operation.

Assuming inner maximization problem's feasible region is not empty, we can use the following reformulation to cast bilevel MFNIP into single level minimization linear program. Let α and β denote dual variables associated with the flow conservation and arc capacity constraints, respectively. Note that dual program of a maximum flow problem finds minimum cut set of the network. Then MFNIP's final reformulation is as follows.

MFNIP-D Single Level Formulation [35]**:**

$$\min \sum_{i\in A}\sum_{j\in A} u_{ij}(1-\gamma_{ij})\beta_{ij} \qquad (3.1.2.7)$$

$$s.t. \quad \alpha_i - \alpha_{ij} + \beta_k \ge 0 \qquad k = (i,j) \in A\backslash\{(t,s)\} \qquad (3.1.2.8)$$

$$\alpha_t - \alpha_s \ge 1, \qquad (3.1.2.9)$$

$$\beta \ge 0 \qquad (3.1.2.10)$$

$$\gamma \in \Gamma \qquad (3.1.2.11)$$

This problem is not linear due to the bilinear terms $\gamma_{ij}\beta_{ij}$ in (3.1.2.7). However, the dual variables α and β can be restricted to be binary values. Because changing the right-hand side of the associated constraints to dual variables in the primal problem of MFNIP leads to the change at most one unit of the maximum flow.

At optimality;

$$\alpha_i = \begin{cases} 1, & if\ node\ i \in N\ is\ on\ the\ sink\ side\ of\ the\ mimum\ cut, \\ 0, & otherwise \end{cases}$$

$$\beta_{ij} = \begin{cases} 1, & if\ arc\ (i,j) \in A\ is\ in\ the\ minimum\ cut, \\ 0, & otherwise \end{cases}$$

Therefore, this bilinear model can be converted to a linear mixed integer program by substituting $\gamma_{ij}\beta_{ij}$ with a single variable θ_{ij} and adding extra constraints $\theta_{ij} \leq \gamma_{ij}$, $\theta_{ij} \leq \beta_{ij}$, $\theta_{ij} + (1 - \beta_{ij}) \geq \gamma_{ij}$, and $\gamma_{ij} \geq 0$. In fact, last two constraints are useless because bilinear terms appear only in the objective (3.1.2.7) with negative signs [31].

A Basic Example for the MFNIP [36].

Consider the network in Figure 3.1.2.1 where *capacity levels* of arcs written on top of them. NU wishes to maximize flow on this network and NI attempts to minimize the maximized flow. Assume that *interdiction budget* is just enough to disrupt one arc.

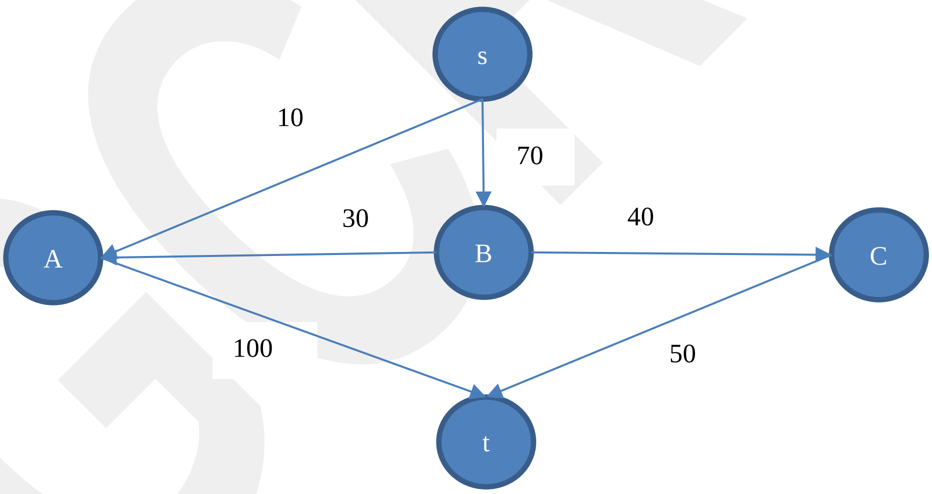


Figure 3.1.2.1. Example Network for the MFNIP Example

Capacity levels of arcs:

$u_{sA} = 10,\ u_{sB} = 70,\ u_{At} = 100,\ u_{BA} = 30,\ u_{BC} = 40\ and\ u_{Ct} = 50.$

For no interdiction case; the example is a simple Maximum Flow Problem. The solution in this case is given by same notation to the MFNIP:

$\mathrm{x}_{\mathrm{sA}} = 10,\ \mathrm{x}_{\mathrm{sB}} = 70,\ \mathrm{x}_{\mathrm{At}} = 40,\ \mathrm{x}_{\mathrm{BA}} = 30,\ \mathrm{x}_{\mathrm{BC}} = 40$ and $\mathrm{x}_{\mathrm{Ct}} = 40$ (in units).

Moreover, NU's maximum *amount of flow* is 80 units (can also be found as the minimum capacity cut which is $u_{sA} + u_{sB} = 80$)

Now, assume that a network interdictor wishes to attack to the network depicted in Figure 3.1.1. to minimize the maximum flow of NU. Moreover, he has a budget for only one arc interdiction.

After NI's move, solution network is shown in Figure 3.1.2.2. NI interdicts *arc(s, B)* by decreasing capacity of the arc to zero $u_{sB} = 0$ with its decision variable, $\gamma_{sB} = 1$.

In this resulted network, NU can move its flow only arcs *arc(s, A) and arc(A,t) with amount units*, $x_{sA} = 10, x_{At} = 10$. Because, *arc(s,B)* can not carry anymore.

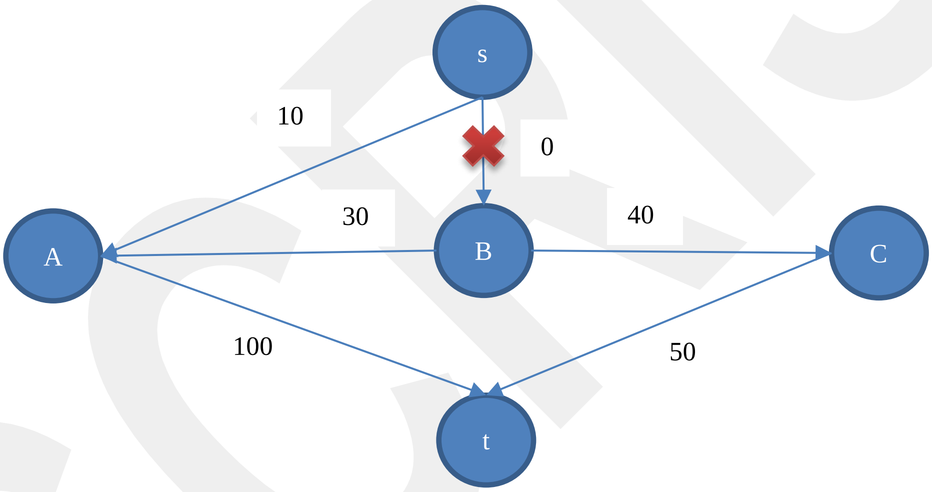


Figure 3.1.2.2. Solution Network for the MFNIP Example

NU's maximum amount of flow is *10* units *after interdiction*. NI cuts off the maximized flow with one arc interdiction by 87.5%.

A stochastic version of MFNIP [37] is studied by Cormican et al. (1998) with a two-stage Stochastic program. In stochastic maximum-flow interdiction, both arc capacities and success rate of NI can be random. Therefore, NI's goal is to minimize the expected value of the maximum flow. The model has real-world applications such as interdicting illegal drug transportation and reducing the effectiveness of a military force while it is moving materiel, troops, and information, through a network in wartime.

Akgün et al. (2011) develop a model for *multi-terminal maximum-flow network interdiction problem* (MTNIP) [38]. MTNIP is a generalized version of the MFNIP. The problem context in MFNIP is the same as the one in MTNIP except that the interdictor tries to minimize the maximum flow from a source node *s* to a sink node *t* instead of among three or more groups of nodes. That is, MFNIP is a special case of MTNIP with two groups of nodes.

3.1.2.2. Shortest Path Interdiction

Consider a directed graph $G = (N, A)$ with a node set and an arc set, N and A, respectively. In shortest path interdiction problem, network user wishes to traverse from source node $s \in N$ to terminus node $t \in N$ by using the shortest path. NI interdicts some subset of A to maximize the minimum/shortest path of NU. Interdiction is done by increasing the length of arc $(i, j) \in A$ from c_{ij} to $c_{ij} + d_{ij}$. This is so called penalty cost in interdiction literature. Because NI penalizes NU if those interdicted arcs are used. Therefore, d_{ij} values must be enough incentive for NU not to traverse them in case of interdiction. Israeli and Wood (2012) is firstly studied the *shortest path interdiction problem* (MXSP) [39]. MXSP is modeled as follows.

Indices and Sets:

$i \in N$ and $j \in N$ represent nodes of the network G.

$(i, j) \in A$ defines an arc between node *i* and node *j*.

$FS(i)$ is set of arcs leaving node *i*.

$RS(i)$ is set of arcs entering node *i*.

Data:

R interdiction budget

r_{ij} required resource to NI interdicting one arc

c_{ij} original length of arc *(i,j)*; $c_{ij} \geq 0$

d_{ij} additional length (penalty cost) if arc *(i,j)* is interdicted; $d_{ij} \geq 0$

NI's Decision Variables

$x_{ij} = 1$ if arc *(i,j)* is interdicted and 0 otherwise.

NU's Decision Variables:

$y_{ij} = 1$ if arc *(i,j)* is traversed in the shortest path and 0 otherwise.

MXSP Formulation, [39]:

$$\max_{x \in X} \min_{y} \sum_{(i,j)\in A} (c_{ij} + d_{ij}x_{ij})y_{ij} \qquad (3.1.2.12)$$

$$s.t. \quad \sum_{(i,j)\in FS(i)} y_{ij} - \sum_{(i,j)\in RS(i)} y_{ji} = \begin{cases} 1, i = s \\ 0, i \in N\backslash\{s,t\} \\ -1, i = t \end{cases} \qquad (3.1.2.13)$$

$$y_{ij} \geq 0, \quad (i,j) \in A \qquad (3.1.2.14)$$

$$where\ \mathrm{X} \equiv \left\{x_{ij} | x_{ij} \in \{0,1\}, \forall (i,j) \in A, \sum\nolimits_{(i,j)\in A} r_{ij}x_{ij} \leq R\right\}$$

Objective (3.1.2.12) increases the length of arc *(i,j)* when it is interdicted $(y_{ij}) = 1$. Constraints (3.1.2.13) are NU's flow conservation restrictions. Note that NI's variables, x_{ij} appear only in the objective function. This time, right-hand side of the problem is free from NI.

General-purpose solvers for MIPs can not solve MXSP. However, inner minimization problem is an LP and therefore we can fix interdictor's variable $x \in X$, then take the dual of the inner LP and release *x*. Letting π denote the dual variables corresponding to flow conservation constraints, MXSP can be cast into a single maximization linear program as follows.

Dual variables:

π_t denotes the length from source node *s* to node *i*.

MXSP-D Single Level Formulation, [39]**:**

$$\max \sum_{x,\pi} \pi_t - \pi_s \qquad (3.1.2.15)$$

$$s.t. \qquad \pi_j - \pi_i - d_{ij}x_{ij} \le c_{ij} \qquad (i,j) \in A \qquad (3.1.2.16)$$

$$\pi_s = 0 \qquad (3.1.2.17)$$

$$x \in X \qquad (3.1.2.18)$$

The resulting MXSP-D program is a linear program. It can be solved by branch and bound algorithm theoretically. However, this approach may fail because LP relaxation of the program can be weak due to penalty cost d_{ij} variables are large relatively to the original lengths, c_{ij}. Therefore, an algorithm based on Benders decomposition [39] is proposed to solve MXSP and tighter formulations so called super valid inequalities are generated. In the same study, researchers also consider a three-stage fortification game with two players, NU and NI. Also, they study stochastic interdiction on shortest path problem in which NI monitors the network by controlling some arcs which are chosen by probabilistic methods, while NU tries to traverse from *s* to *t* without being detected by NI. In this case, NU searches for a *reliable path* rather than the shortest path.

3.1.2.3. Minimum Cost Flow Interdiction

Consider a directed graph $G = (N, A)$ with a node set and an arc set, N and A, respectively. In the minimum cost flow interdiction, NU creates flows to minimize costs while NI makes interdiction plans to maximize the NU's minimum value. A generic minimum cost flow interdiction problem is as follows [31]:

Indices:

$i \text{ and } j \in N$ represent nodes of the network G.

$(i\,,j)$ represents the arc between nodes *i* and *j*.

$FS(i)$ is set of arcs leaving node *i*.

$RS(i)$ is set of arcs entering node *i*.

Data:

R interdiction budget

r_{ij} required resource to NI interdicting one arc

c_{ij} is the cost of moving one unit of flow on the arc *(i, j)*.

d_i is the supply (if positive) or the demand (if negative) present at node i.

u_{ij} is the capacity level of arc *(i, j)*.

NU's Decision Variables:

y_{ij} is the amount of flow leaving from node i and entering to node j

NI's Decision Variables:

$x_{ij} = 1$ if arc *(i, j)* is interdicted, 0 otherwise.

Minimum Cost Flow Interdiction Formulation:

$$\max_{x \in X} \; min \sum_{(i,j) \in A} c_{ij} y_{ij} \tag{3.1.2.19}$$

$$s.t. \quad \sum_{(i,j) \in FS(i)} y_{ij} - \sum_{(i,j) \in RS(i)} y_{ji} = d_i \qquad i \in N \tag{3.1.2.20}$$

$$y_{ij} \leq u_{ij}(1 - x_{ij}) \qquad (i,j) \in A \tag{3.1.2.21}$$

$$where\ \mathrm{X} \equiv \left\{ x_{ij} | x_{ij} \in \{0,1\}, \forall (i,j) \in A, \sum\nolimits_{(i,j) \in A} r_{ij} x_{ij} \leq R \right\}$$

Note that the formulation is similar to MFNIP except that the interdictor's variable is in the objective function (3.1.2.12). Let α and β denote dual variables associated with the flow conservation and arc capacity constraints, respectively.

Minimum Cost Flow Interdiction-Dual Single Level Formulation:

$$\max \sum_{i \in N} d_i \alpha_i - \sum_{(i,j) \in A} u_{ij}(1 - x_j) \beta_{ij} \tag{3.1.2.22}$$

$$s.t. \quad \alpha_i - \alpha_j - \beta_k \leq c_k \qquad k = (i,j) \in A \tag{3.1.2.23}$$

$$\beta \geq 0 \tag{3.1.2.24}$$

$$x \in X \tag{3.1.2.25}$$

In MFNIP, we could restrict dual variables to be binary values. However, the same rule does not apply for minimum cost flow interdiction problem. Since change in the objective function value per unit flow change is dependent on the cost vector *c*. See [40] for exact algorithms for multicommodity flow network interdiction problem which is similar to minimum cost flow interdiction.

Various optimization algorithms can solve network interdiction models. However, usually, there are two solution approaches are common in the literature. One is casting bilevel program into the single level structure, then facilitate well-known algorithms as in MFNIP [35]. Other is Benders decomposition that separate two level program into master and subproblems and solve them sequentially as in MXSP [39]. See [41], [42] for reviewing different solution approaches.

3.2. Facility Interdiction Problem

Network interdiction models reflect disruption effects via arcs of the network. Furthermore, node disruption so called facility interdiction is another aspect of network interdiction problems. Scaparra and Church (2015) address three main questions for facility interdiction problem [3].

(1) To search for the critical parts of a system; which facilities cause the most severe damage when they are removed from the network?

(2) To protect the network from a disturbance risk; which facilities would be fortified against a disaster?

(3) As to settling facilities in the occurrence of disturbance; how can the system be resilient when disrupted? The optimization models, regarding these 3 questions, are classified into three categories:

1) *Interdiction models* identify the critical facilities to a system.
2) *Protection models* choose the necessary facilities that should be fortified against disruption.
3) *Design models* are used to create resilient networks considering of possible future disruptions.

Here, we do not review the whole facility interdiction literature. Instead, we consider the generic type of interdiction formulations. See Scaparra and Church (2015)'s review of facility location problems under disruption [3].

3.2.1. Interdiction Models

Church et al. (2004) interpret interdiction game [43] in which NI can disrupt r out of p facilities. Interdicted facilities are removed from the network and NU assigns free nodes to its closest facility. Researchers consider interdiction on p-median and maximal covering problems. Here, we only review interdiction on the p-median problem so called r-*interdiction p-median problem* (r-IMP). r-IMP is modeled as follows.

Indices and Sets:

$I =$ Set of potential locations for the facilities, indexed by i.

$J =$ Set of customers, indexed by j.

$F =$ Set of facilities in an existing system.

Data:

$d_j =$ Demand of customer j.

$c_{ij} =$ Unitary cost for serving customer j from facility i.

$p =$ Number of facilities to be located.

$r =$ Number of facilities to be interdicted.

Decision Variables:

$$y_i = \begin{cases} 1, if\ a\ facility\ is\ located\ at\ site\ i \\ 0, otherwise \end{cases}$$

$$s_i = \begin{cases} 1, if\ a\ facility\ is\ interdicted\ at\ site\ i \\ 0, otherwise \end{cases}$$

$$x_{ij} = \begin{cases} 1, if\ the\ demand\ of\ customer\ j\ is\ supplied\ from\ facility\ i \\ 0, otherwise \end{cases}$$

Letting $T_{ij} = \{k \in F|\ d_{kj} > d_{ij}\}$ is the set of existing sites (not including *j*) that are at least farther than *i* to the demand *j*. We call demand points which lost their supplier facility due to interdiction as *free demand points*.

r-IMP Formulation [43]**:**

$$\max \sum_{i \in N} \sum_{j \in F} d_j c_{ij} x_{ij} \qquad (3.2.1)$$

$$s.t. \qquad \sum_{j \in F} x_{ij} = 1 \qquad j \in F \qquad (3.2.2)$$

$$\sum_{j \in F} s_j = r \qquad (3.2.3)$$

$$\sum_{k \in T_{ij}} x_{kj} \le s_i \qquad i \in F, j \in J \qquad (3.2.4)$$

$$x_{ij} \in \{0,1\} \qquad i \in F, j \in J \qquad (3.2.5)$$

$$s_i \in \{0,1\} \qquad i \in F \qquad (3.2.6)$$

The objective (3.2.1) maximizes demand-weighted distance under the impact of interdiction of *r* facilities. Constraints (3.2.2) ensure that demand points are assigned to a facility. Constraint (3.2.3) interdicts *r* facilities out of optimal *p* facilities. (3.2.4) constraints ensure that if a demand point loses its facility, it is assigned to a non-interdicted facility. Constraints (3.2.5) and (3.2.6) define binary decision variables.

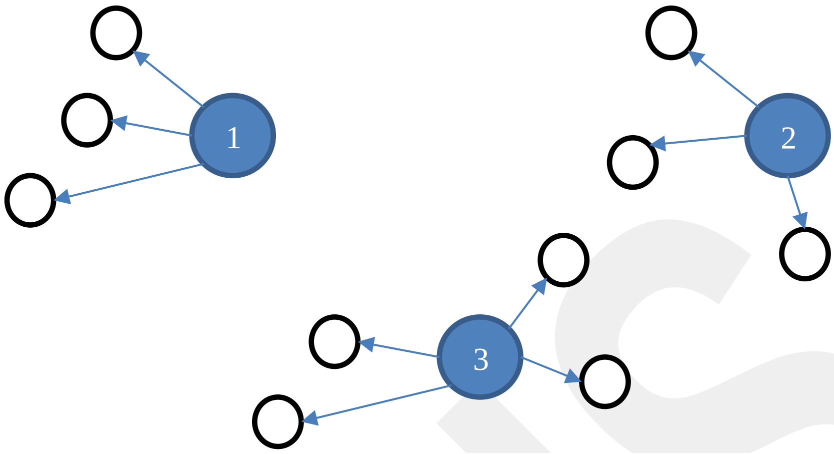


Figure 3.2.1 Facility - Demand Point Assignment

For example, Figure 3.2.1 shows a simple solution for a p-median problem of NU with three facilities. Assume that NI attacks up to 1 out of 3 facilities and interdicts third facility. Then free demand points previously assigned to the interdicted hub are assigned to first and second hubs regarding closest facility criteria as shown in Figure 3.2.2.

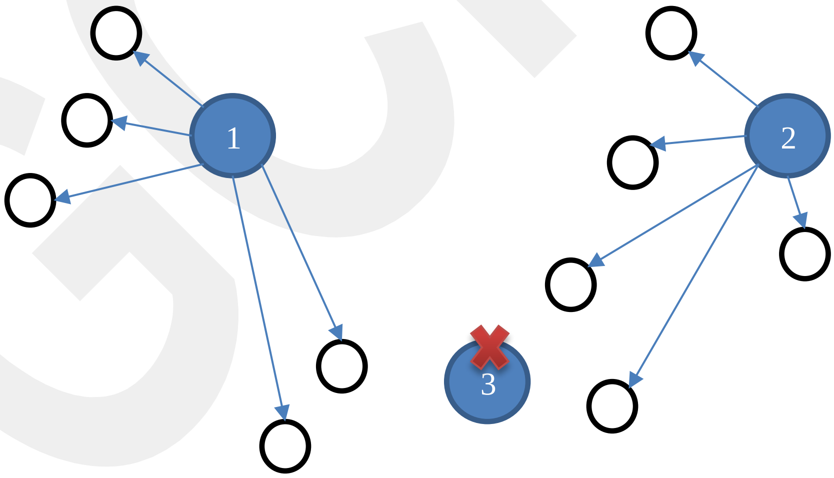


Figure 3.2.2. Facility - Demand Point Assignment after Interdiction

Note that unlike traditional network interdiction problems, r-IMP has a single level formulation. Therefore, the problem can be solved by general-purpose optimization solver software.

r-IMP do not limit the capacity of facilities that means survivor facilities can supply free demand points even after interdiction. However, this may not be applied to real-world problems. Therefore, Scaparra and Church (2012) develop capacitated version of r-IMP [44]. Moreover, Losda et al. (2012) assume that success of interdiction is probabilistic and this probability depends on the magnitude of the disruption, therefore, they introduced a two-stage stochastic program for r-IMP [45].

3.2.1.1. Hub Interdiction Models

Lei (2013) develops an r-interdiction problem on hub-and-spoke network structure so-called *Hub Interdiction Model* (HIM) [46]. HIM uses the same interdiction concept as in r-IMP, only differs at considering p-hub median interdiction problem. NI interdicts *r* hub facilities out of existing *p* hubs. O/D flows which previously allocated by interdicted hubs reroute over least-cost paths including survivor hubs. That is, residual hubs allocate free O/D flows after interdiction regarding least-cost route criteria. HIM is modeled as follows.

Indices and Sets:

$G = (N, A)$ is a complete graph

A is the set of edges in the hub-and-spoke network

N is the set of nodes in the hub-and-spoke network

$i \in N$ represents an origin node.

$j \in N$ represents a destination node.

$k \in N$ represents the hub node in the collection leg of a O/D path.

$m \in N$ represents the hub node in the distribution leg of a O/D path.

Data:

α is the discount factor on hub arcs.

d_{ij} is the distance between *node i* and *node j*.

$C_{ijkm} = d_{ik} + \alpha(d_{km}) + d_{jm}$ is flow cost over a O/D path.

Decision Variables:

W_{ij} is the amount of flow between *node i* and *node j*.

Z_{ik} is equal to *1* if *node i* is allocated by *hub node k*, *0* otherwise.

X_{ijkm} is demand service level from *origin i* to *destination j* goes through *hubs k* and *m*, and takes value between *0* and *1*.

y_i if hub *k* is not interdicted, and *0* otherwise.

The following set describes the relative order of costs for different routes for any given O/D pair:

$$E_{ijkm} = \left\{ \begin{matrix} (q,s) |\ d_{iq} + \alpha d_{qs} < d_{ik} + \alpha d_{km} + d_{mj}\ or\ d_{iq} + \alpha d_{qs} + d_{sj} = d_{iq} + \alpha d_{km} + d_{mj}\ and \\ (q < k\ or\ q = k\ and\ s < m) \end{matrix} \right\}$$

This set involves pairs of hub facilities that generate routes with strictly lower costs than route *i-k-m-j*.

HIM Formulation [46]:

$$\max \sum_i \sum_j \sum_k \sum_m W_{ij} C_{ijkm} X_{ijkm} \tag{3.2.7}$$

$$s.t. \quad \sum_k \sum_m X_{ijkm} = 1 \qquad i,j \in N \tag{3.2.8}$$

$$\sum_k X_{ijkm} = y_m \qquad i,j \in N;\ m \in F \tag{3.2.9}$$

$$\sum_m X_{ijkm} = y_k \qquad i,j \in N;\ k \in F \tag{3.2.10}$$

$$\sum_k y_k = p - r \tag{3.2.11}$$

$$\sum_{(q,s) \in E_{ijkm}} X_{ijqs} + X_{ijkm} \geq y_k + y_m - 1 \qquad i,j \in N;\ k,m \in F \tag{3.2.12}$$

$$y_k \in \{0,1\} \tag{3.2.13}$$

$$0 \leq X_{ijkm} \leq 1 \tag{3.2.14}$$

The objective function (3.2.7) maximizes total flow cost after interdiction. Constraints (3.2.8) ensure that each O/D flow can only be assigned to one pair of hubs. Constraints (3.2.9) and (3.2.10) maintain that O/D flows can be routed only via survivor hubs after interdiction. Constraint (3.2.11) interdicts *r* hub facilities out of existing *p* hubs that also means *p*-*r* hubs be kept after interdiction. Constraints (3.2.12) ensure that if both hub *k* and hub *m* are kept open and if no route has a lower cost than route *i*-*k*-*m*-*j* (i.e. $\sum_{(q,s)\in E_{ijkm}} X_{ijqs} = 0$), then route *i*-*k*-*m*-*j* must be assigned to the flow between origin *i* and destination *j*. Without this type of costraint, the model assigns all O/D flows to the greatest cost routes.

Unlike classic interdiction games as in Sect. 3.1. HIM has a single-level formulation which is solved by Branch and Bound algorithm. NI attacks up to *r* hubs out of existing *p* hub facilities of NU. NI wishes to maximize flow costs with the objective function (3.2.7), while NU assign O/D flows to least cost routes with constraints (3.2.12). That is, the objective of HIM identifies most critical hubs and interdict them to maximize of NU's flow cost. Resulted network after interdiction will have *p*-*r* hub facilities. NU will assign all O/D flows to least routes including those survivor facilities.

3.2.2. Protection Models

Protection problems attempt to find hubs that need fortification to avoid the worst-case scenario. Protecting only most critical facilities determined by a r-IMP solution may lead to the wrong solution. Therefore, Scaparra and Church (2008) expands r-IMP to a two-stage game [47] to include a fortification stage, where the network user moves first to protect a subset of hub facilities from the network interdictor. The interdictor must then work on all attacks to those unprotected hub facilities.

Indices and Sets:

I = Set of potential locations for the facilities, indexed by i.

J = Set of customers, indexed by j.

F = Set of facilities in an existing system.

Data:

d_j = Demand of customer j.

c_{ij} = Unitary cost for serving customer j from facility i.

p = Number of facilities to be located.

b = Number of facilities to be protected.

Decision Variables:

$$s_i = \begin{cases} 1, if\ a\ facility\ is\ interdicted\ at\ site\ i \\ 0, otherwise \end{cases}$$

$$z_i = \begin{cases} 1, if\ a\ facility\ is\ located\ at\ i\ is\ protected \\ 0, otherwise \end{cases}$$

$$x_{ij} = \begin{cases} 1, if\ the\ demand\ of\ customer\ j\ is\ supplied\ from\ facility\ i \\ 0, otherwise \end{cases}$$

r-IMPF Formulation [48]**:**

$$\min H(z) \qquad (3.2.15)$$

$$s.t. \qquad \sum_i z_i = b \qquad (3.2.16)$$

$$z_i \in \{0,1\} \qquad i \in F \qquad (3.2.17)$$

where

$$H(z) = max \sum_i \sum_j d_j c_{ij} x_{ij} \qquad (3.2.18)$$

$$s.t. \qquad s_i \leq 1 - z_i \qquad i \in F \qquad (3.2.19)$$

(3.2.2) - (3.2.6)

The objective function (3.2.15) minimizes the maximum flow cost generated by NI in the (3.2.18). Constraint (3.2.16) protects b hubs from the following interdiction in the inner level which is simply the r-IMP formulation with additional constraints. NI cannot interdict protected facilities due to restrictions by constraints (3.2.19) which link interdiction variables of NI and protection variables of NU.

Scaparra and Church (2008) observe that at least one of the protected facilities must also be a critical hub [47]. Therefore, they suggest implicit enumeration for r-IMPF rather than total enumeration.

Based on r-IMPF; a stochastic version is modeled by Liberatore et al. assuming the exact value of *r* is not known [49]. Moreover, Bricha and Nourelfath (2013) expands the problem for a realistic environment that success of protection of a facility is probabilistic [50].

3.2.3. Design Models

Interdiction and protection models are useful to identify critical and must-protected facilities in existing network structures. Design models attempt to build robust network systems by considering potential disruptions at network design process. These models identify alternative plans for network designers in the case of future disruption.

O'Hanley and Church (2011), and Parvaresh (2012) develop bi-level interdiction games [51], [52] on maximal covering problems considering a risk-averse network designer so that interdictor's attack is reflected in the worst-case.

Snyder and Daskin (2006) work on the reliability of p-median and fixed charge facility location networks [53] considering facility failures. They model reliability problems in the perspective of a risk-neutral network designer that assumes facilities to fail at random.

Next Chapter 4 introduces Flow-based p-Hub Median Interdiction Problem which is considered to be in design interdiction models.

Chapter 4

FLOW-BASED p-HUB MEDIAN INTERDICTION PROBLEM

We examine the interdiction of an uncapacitated hub-and-spoke network. The node set of the network represent origins and destinations, and some nodes can serve as hub facilities to transfer service flows between origin and destination (O/D) pairs (see Chapter 2 for a detailed explanation of the hub-and-spoke network structure). On that network, the problem we consider takes the perspectives of two decision makers, e.g. a network user (NU) and a network interdictor (NI). The NU wishes to minimize transportation cost of service flows across the network by facilitating *p* hub facilities, and the NI attempts to worsen the NU's objective by disrupting NU's facilities. NI has limited resources to implement an attack plan against NU. NI's attack locations are restricted to the node set of the network. Damage is inflicted on each node by removing its hub functionality that means an interdicted node can be both origin and destination point, however, cannot be a hub facility. We refer to this problem as the *Multi-Commodity Flow-based p-hub Median Interdiction Problem* (MCFPIP).

MCFPIP can be modeled with a game theoretical approach since it is a genuinely two-stage and two-player game. NI makes the first move by attacking nodes of the network. NU makes the following move on the resulted network after interdiction. Hence, hub interdiction problem is a truly Stackelberg game in which the leader/interdictor moves first and the follower/operator acts against that move [4], [5]. If there is no human interdictor, natural threats take the role of NI in worst-case analyses. That is, “Murphy’s Law”, the pessimistic approach that the worst event will occur, change the network structure [31].

4.1. Background

The motivation of studying of MCFPIP is due to the extensity of disruption events on hub-and-spoke transportation and telecommunication networks. Like every system, hub-and-spoke networks are also open to numerous possibilities of disruptions (For real-world disruption examples; see Chapter 1 and Chapter 3). Hubs location designs are essential because a disturbance on hubs may cause the network system to fail. Therefore, the reliability of hubs against disruptions is a key criterion to design a successful network system. MCFPIP provides valuable insights into identifying critical hub facilities.

Although the analysis of disruptions on hub facilities is of great importance, only a few studies have focused on this issue. In previous studies on hub interdiction problem; existing facilities of hub network systems are considered as explained in Sect. 3.2.1. These problems assume that NI interdicts *r* out of predetermined *p* hub facilities and then NU sustains its operations with remaining *p*-*r* facilities.

MCFPIP incorporates the risks of possible interdiction operations in the initial design of a hub-and-spoke network system by identifying alternative hub location strategies which are both cost-efficient and robust to external disruptions. Unlike previous hub interdiction studies, MCFPIP assumes that NI interdicts most critical facilities of the network and NU can continue its activities on the resulted network after interdiction with *p* hub facilities. Note that these *p* hub facilities are not the optimal solution set of NU in the initial network.

Let’s explain the difference between *facility interdiction problems* examined in Sect. 3.2. MCFPIP with the Example 4.1.

Example 4.1. (with arbitrary values)

Consider a complete network with 9 nodes. Suppose NU wishes to locate 4 hubs optimally such transportation cost between all node pairs is minimized. We assume that NU finds out that nodes *1, 4, 5 and 7* are optimal hubs.

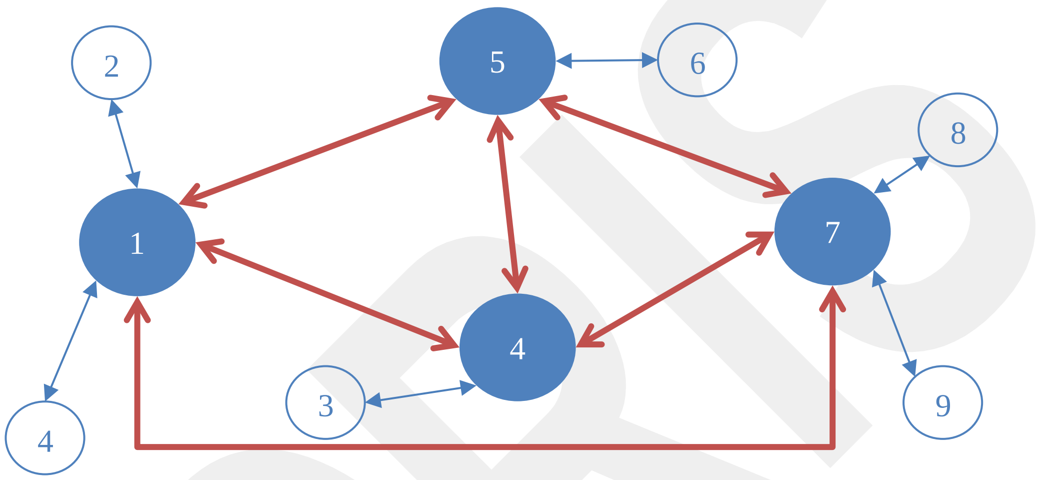


Figure 4. 1. No interdiction case for Example 4.1.

In this case, we assume that NU's objective value to be *K* units. This solution is shown in Figure 4.1.

Now, suppose that an NI attacks the network depicted in Figure 4.1. with its two-node interdiction resources. HIM [46] solves this problem by interdicting two existing optimal hubs out of three hubs which are nodes *1, 4, 5, and 7* and assign free origin and destinations to survivor hub facilities. Assuming HIM interdicted nodes *4 and 5*, free nodes *3 and 6* are allocated to closest remaining hub facilities *1 and 5*, respectively as shown in Figure 4.2. Note that a hub facility can have its own supply and demand. Interdiction of a hub facility is not removing it from the network. So, interdicted facilities 4 and 5 can be allocated by hub node 1 and 7, respectively as in Figure 4.2.

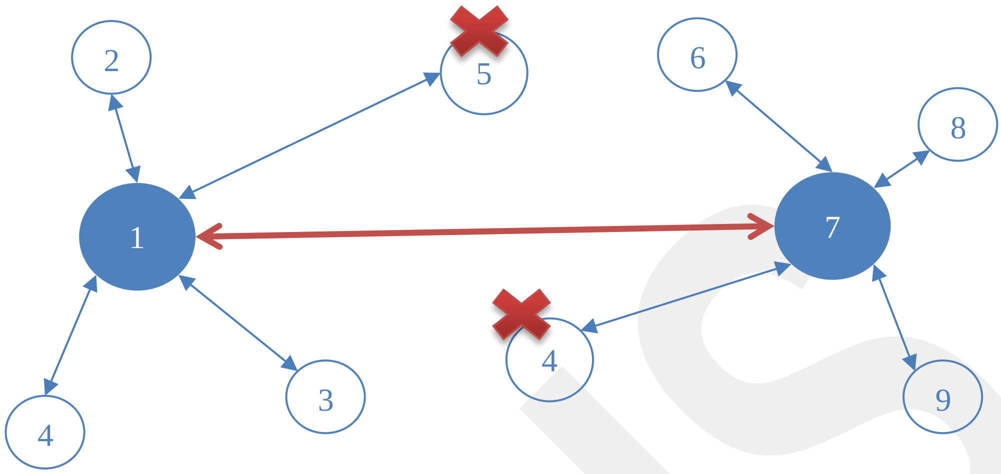


Figure 4. 2. HIM Solution to Example 4.1. with 2-node interdiction case

Let MCFPIP solve the example 4.1. for the same NU and NI with the same objectives and resources. Assume that MCFPIP interdicts two nodes *6 and 7*. Node 7 was already a hub facility in the initial solution. Note that node *6* was not a hub facility, but it was a candidate hub. However, after interdiction, node *6* cannot be a hub anymore since it has lost hub functionality property. Therefore, NU locates *4* hubs at nodes *1, 3, 4 and 5* in the resulted network after interdiction as shown in Figure 4.3.

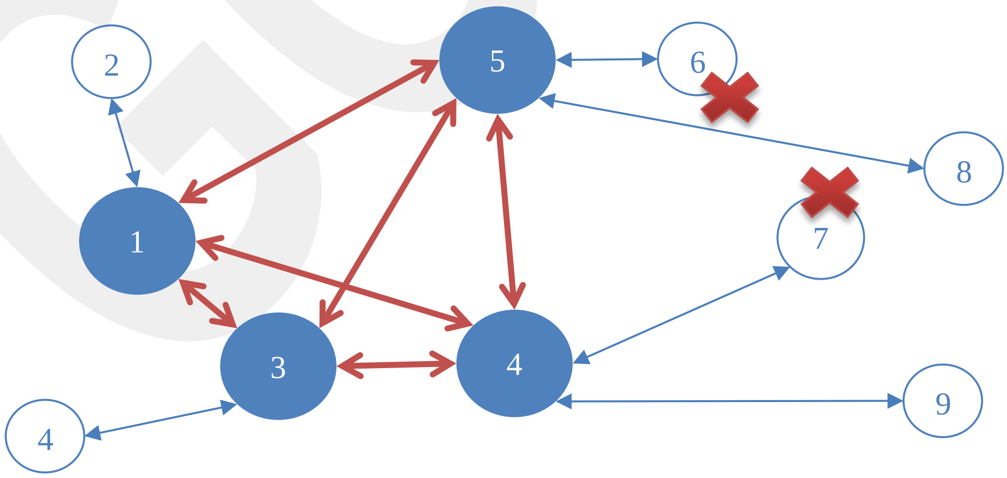


Figure 4. 3. MCFPIP Solution to Example 4.1. with 2-node interdiction case

Lei (2013)'s HIM [46] and studies based on r-IMP given in Sect. 3.2. analyzes the worst-case scenario in which NI attacks most critical facilities and cripples NU's network at the worst level. NU must leave *r* facilities and continue with *p-r* facilities. This scenario can apply to existing network systems assuming the system cannot recover itself in a short time. However, if NU designs a new network structure, then it needs a new tool to evaluate critical infrastructure of the network system and should prepare an alternative plan to create a robust network.

4.2. Model Development

Interdiction problems are associated with bilevel programming (also known as a two-level and hierarchical optimization) if leader's and follower's objectives are different. MCFPIP is a two-stage game between two players who have different objectives. Therefore, we model MCFPIP in a bilevel program which is appropriate to model two-stage Stackelberg game [5]. In this interdiction game, NI and NU sequentially make decisions in a noncooperative manner. The bilevel program includes of decision variable sets of NU and NI. Inner level constraints NU's decision set to be a solution of optimization problem of given NI's decision variable from the upper level. The upper level is an integer program due to interdictor's variables, and the inner level is a mixed integer program of the flow-based p-hub median problem of [23]. Therefore, flow-based p-hub median interdiction bilevel integer program we propose is strongly NP-hard [31].

Multicommodity flow-based p-hub median interdiction problem (MCFPIP) is modeled as a bilevel program as follows:

Indices and Sets:

$G = (N, A)$ is the underlying hub-and-spoke network (see Sect. 2.5 for detailed explanation)

H is the set of potential hub facilities indexed by *i*.

S is the demand generating node set indexed by *k*.

A_0 is the set of arcs of three layered network explained in Sect. 2.5.1 and this set indexed by *a*.

β is the set of nodes explained in detail in Sect. 2.5.1

F_{β}^{out} is the forward star of a node β consisting of arcs whose tail is β.

F_{β}^{in} is the inward star of node $\beta \in N_0$ consisting of arcs whose heads are β.

C denotes total interdiction budget for interdiction.

Data:

l_a is the length of the arc *a*. (see Sect. 2.5.1 for detailed formulation)

c_i is the amount of resource to interdict hub *i*.

M_i, penalty cost (in length) added to site *i*.

W_k is the amount of supply for commodity *k*

Network Interdictor's Decision Variables:

t_i is equal to 1, if a hub facility is interdicted at *i*, 0 otherwise.

Network User's Decision Variables:

y_i is equal to 1, if a hub facility is located at site *i*, 0 otherwise.

X_{ak} is the amount of flow on arc *a* for commodity *k*.

MCFPIP Formulation:

$$max_{\, t\epsilon T_I}$$

$$\min Z^* = \sum_{k\epsilon S} \sum_{a\epsilon A_0} l_a X_{ak} + \sum_{i\in H} (t_i M_i) y_i \tag{4.1}$$

$$where \quad T_I = \left\{ t: \sum_{i\in H} c_i t_i \leq C, t_i \in \{0,1\}, \forall i \in H \right\}$$

$$s.t. \qquad \sum_{a\in F_{\beta}^{out}} X_{ak} - \sum_{a\in F_{\beta}^{in}} X_{ak} = b_{\beta k} \quad \beta \in (N_1 \cup N_2 \cup N_3), \; k \in S \tag{4.2}$$

$$\sum_{i\in H} y_i = p \tag{4.3}$$

$$X_{(1i,2i)k} \leq W_k y_i \quad i \in H, k \in S \tag{4.4}$$

$$X_{(2i,3i)k} \leq W_k y_i \quad i \in H, k \in S \tag{4.5}$$

$$X_{ak} \geq 0 \quad a\epsilon A_0, k \in S \tag{4.6}$$

$$y_i \epsilon \{0,1\} \quad i \in H \tag{4.7}$$

Objective (4.1) maximizes the minimum flow cost. For fixed t_i values, the remaining model is the NU's model. When a located hub *i* is interdicted, the second term in the objective function adds a high penalty for hub *i* to the objective function. This prevents NU from using this interdicted hub. The inner minimization problem is uncapacitated flow-based multiple allocation p-hub median problem (G-MApHMP) with the constraint set (4.2 – 4.7) (see the details in Sect. 2.5). Constraints (4.2) are well-known flow conservation restrictions. Constraint (4.3) locates *p* hub facilities over the network. Constraints (4.4) and (4.5) link first graph layer to second layer and second layer to third level, respectively. T_I is the interdiction resource budget set that restricts interdiction operations.

Another way to model this problem is to disconnect hubs from level 1 and level 3 of the three-layered structure of G-MApHMP. By this method, we need to add interdiction variables into constraints (4.4) and (4.5). However, [54] states that the NI's (upper-level) decision variables on the right-hand side of the constraint set of the NU's (inner – level) make the problem harder to solve due to non-convexity of the inner level. The NI's decision variable can transform the original one into non-convex. Hence, maximizing (upper-level objective) a non-convex function is a very hard problem to solve. Therefore, the researcher suggests for a shortest path interdiction problem adding penalty cost in the objective function. Our problem also has a similar hierarchical structure with the shortest path interdiction problem, MXSP. Hence, we have been using penalty cost formulation.

4.3. A Decomposition Based Solution Procedure

MCFPIP is a bi-level model, and therefore we cannot directly solve it by usually mixed integer programming (MIP) solving techniques. A common solution approach for a multi-level interdiction model is *casting to single level formulation* [35], [40] so that it can be solved by a MIP solution method, i.e. branch and bound algorithm. However, casting to the single level formulation is not reasonable for MCFPIP due to binary integer variables in the constraints (4.4) and (4.5). Another approach is total or implicit enumeration [47].

The other common approach is *decomposition method*. It was used in various network interdiction models [40], [54] Moreover, Wood et al. (2010) state that most algorithms that have been developed for bi-level MIP assume a strong relation between upper- and inner- level part of the original problem [55]. Therefore, the researcher points out the effectiveness of *Benders decomposition* [56] which divides decision variable set into subsets and creates two stages. The first stage is called master problem, and it is solved for a subset of variables. A remaining subset of decision variables is solved by a second stage so-called subproblem.

In our solution procedure, we adopt a decomposition approach similar to Benders decomposition. *Decomposed MCFPIP* is as follows:

Decomposed MCFPIP:

Master Problem ($\widehat{\boldsymbol{Y}}$)

$$z_{\hat{T}} = max_{t \in T}\ z \qquad (4.8)$$

$$\text{s.t.} \quad z \leq l_a X_{ak} + \sum_{i \in H} (t_i M_i) \hat{y}_{\mathrm{i}} \qquad (4.9)$$

where $T_I = \{t \colon \sum_{i \in H} c_i t_i \leq C, t_i \in \{0,1\}, \forall i \in H\}$

Subproblem ($\hat{\boldsymbol{t}}$)

$$min\ Z_{\hat{Y}} = \sum_{k \epsilon S} \sum_{a \epsilon A_0} l_a X_{ak} + \sum_{i \in H} (\hat{t}_{\mathrm{i}} M_i) y_i \qquad (4.10)$$

$$s.t. \sum_{a \in F_\beta^{out}} X_{ak} - \sum_{a \in F_\beta^{in}} X_{ak} = b_{\beta k} \quad \beta \in (N_1 \cup N_2 \cup N_3),\ k \in S \qquad (4.11)$$

$$\sum_{ieH} y_i = p \qquad (4.12)$$

$$X_{(1i,2i)k} \leq W_k y_i \quad i \in H, k \in S \qquad (4.13)$$

$$X_{(2i,3i)k} \leq W_k y_i \quad i \in H, k \in S \qquad (4.14)$$

$$X_{ak} \geq 0 \quad a\epsilon A_0, k \in S \tag{4.15}$$

$$y_i \epsilon\{0,1\} \quad i \in H \tag{4.16}$$

For the master problem:

Objective (4.8) maximizes a function which is constrained by (4.9). Constraints (4.9) are optimality cut added in each solution iteration to the master problem. These cuts identify hub facilities to be interdicted for each iteration regarding penalty cost. In this setting, the hub variable, $\widehat{Y}_{\text{i}}$, is constant since its values are determined by NU in the sub problem. Here, T_I restricts NI by an interdiction budget.

For the subproblem:

The subproblem is truly G-MApHMP. Objective (4.1) minimizes weighted-flow cost over the network with penalization on interdicted hubs. The penalty cost here is an enough incentive for NU not to route flows via hubs that are interdicted in the master problem.

To solve for MCFPIP, we develop Algorithm 1 which is precisely a simple version of the Benders Decomposition algorithm:

Algorithm 1: Benders Decomposition to MCFPIP

Input: An instance of MCFPIP and allowable optimality gap ε.

Output: An Interdiction plan t^* that solves MCFPIP to within ε units of optimality.

Step 0: $\hat{X} \leftarrow \emptyset, \underline{z} \leftarrow -\infty, \bar{z} \leftarrow \infty, \hat{t} \leftarrow 0.$

Step 1: Solve subproblem for solution $\hat{t}$ with objective $z_{\hat{T}}$.

$\hat{Y} \leftarrow \hat{Y} \cup \hat{y}.$

If $\underline{z} < z_{\hat{T}}, t' \leftarrow \hat{t}$ and $\underline{z} \leftarrow z_{\hat{T}}$

Step 2: Solve Master Problem for solution $\hat{t}$ with objective $z_{\hat{Y}}$

$\overline{z} \leftarrow z_{\hat{Y}}$

Step 3: If $\bar{z} - \underline{z} > \in$ then go to Step 1.

Step 4: $t^* \leftarrow t'$, display $t^*$and stop.

The accuracy of the algorithm, as in any Benders Decomposition algorithm, is based on the following observations [54].

1. The subproblem finds an optimal solution against interdictor's decision variable $\hat{t}$. Therefore, $z_{\hat{t}}$ gives a lower bound for interdictor's objective. The bound is finite because of the budget constraint of the interdictor.

2. If the subproblem cannot improve the bounds and lower and upper bounds converge to the desired gap, then the algorithm terminates.

3. The satisfied optimality should occur in finite iterations.

4. When $\hat{Y}$ includes effective hubs, Master problem is equal to MCFPIP. Otherwise, when $\hat{Y} \subseteq Y$ Master problem is a relaxation of the MCFPIP and thus, $z_{\hat{Y}}$ is an upper bound on the interdictor's optimal objective value.

For each iteration of the algorithm, the subproblem finds optimal hubs and gives these hubs to the master problem. Penalty costs are added to the cost on the hubs to be interdicted in the master problem. Then, subproblem determines new hubs from the resulted network after interdiction. These iterations continue until upper and lower bounds of master problem match. This final point is the solution of NU and NI that is the end of the game. Both players find the balance. At the end of the game, NU creates an alternative location plan for hub facilities.

If NU implements this alternative plan on the hub-and-spoke network, NI may not attack because NU can turn to optimal hub locations to reduce its flow cost. However, if NU apply the optimal plan instead of the alternative plan, then the network is open to possible attacks because NI may attack this time to increase NU's flow cost.

Next, Chapter 5 discusses the computations and results of the problem and Chapter 6 concludes the thesis.

Chapter 5

COMPUTATIONS AND RESULTS

5.1. Penalty Cost Calculation

Penalty cost calculation is necessary to run MCFPIP correctly. Because the master problem will interdict the hubs according to penalty cost and subproblem must find this punishment as an incentive not to go through interdicted hubs. In the case of miscalculation, the model may give the wrong solution.

When NI interdicts a hub if NU insists on using the location of this interdicted hub, it must build a new hub next to interdicted one and must accept as much as twice of the demand weighted cost until flows reach both new and interdicted hubs. Therefore, the penalty cost for hub *i*, $(M_i, i \in H)$, is equal to at least transportation cost of all flows coming through hub *i* from beginning node.

Take the example network in Figure 5.1. Penalty cost for hub 3 is:

$$M_3 = length_{123} flow_{123} + length_{13} flow_{13} + \alpha(length_{53} flow_{53} + length_{73} flow_{73})$$

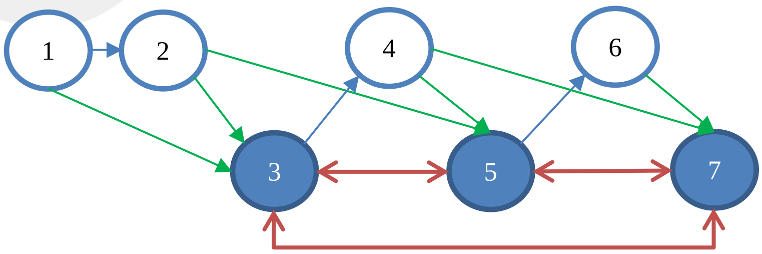


Figure 5.1. Penalty Cost of Formulation Example

5.2. Computation Results

For the implementation of the Algorithm 1, we generate a program on Java API of CPLEX solver on an Intel 4^{th} generation i5 CPU computer with 6 GB RAM and run it on three different data sets that are used in the literature for computation benchmarking:

1-) CAB (Civil Aeronautical Bureau) data set consists of flight passenger data over USA airports. It is named after the number of nodes in it, for example, CAB25 and CAB50. It also includes flow values.

2-) AP (Australia Post) includes package flow over Australia network. The data set consists of 200 nodes. However, we will use 20 nodes version. Flow values are included in the data.

CAB25 and AP20 data are utilized for the experiments, and the results are given in Tables 5.2.1 and 5.2.2, respectively. In the tables, interdictor's budget set, ($|T|$), discount factor (α), NU's last hub facilities (*Selected Hubs)*, additional cost after interdiction in percentage (*Increase*), the number of iterations (*Cuts Count*) and solution times (*Time*) are given.

Table 5.2.1. MCFPIP computations on CAB-25 data

Data Set: CAB-25							
p-hub: 5							
α	**Interdicted Hubs**	**Selected Hubs**	**Hubs before interdiction**	**Total Cost-Interdiction**	**Increase (%)**	**Cuts Count**	**Time (s)**
|T| : 1							
0.3	4	17, 7, 9, 12, 14	17, 4, 7, 12, 14	5431050615.0	5,20	6	154
0.5	12	17, 4, 22, 7, 14	17, 4, 7, 12, 14	6572490579.0	3,57	5	175
0.7	12	17, 4, 22, 7, 14	17, 4, 7, 24, 12	7594774146.0	3,40	4	224
0.9	4	17, 7, 9, 12, 14	1, 17, 4, 7, 12	8269177006.8	2,00	4	426

|T| : 2							
0.3	22, 12	17, 19, 4, 7, 14	17, 4, 7, 12, 14	5628785655.8	9,03	16	319
0.5	4, 12	17, 21, 6, 22, 14	17, 4, 7, 12, 14	6796520995.0	7,11	13	480
0.7	4, 12	17, 19, 21, 6, 14	17, 4, 7, 24, 12	7792435814.1	5,78	8	471
0.9	4, 12	1, 17, 22, 9, 11	1, 17, 4, 7, 12	8370050507.2	3,25	11	870
|T| : 3							
0.3	19, 22, 12	17, 4, 7, 8, 14	17, 4, 7, 12, 14	6113339174.0	18,41	37	671
0.5	19, 22, 12	17, 4, 7, 8, 14	17, 4, 7, 12, 14	7068125636.0	11,39	33	953
0.7	19, 22, 12	17, 4, 7, 8, 24	17, 4, 7, 24, 12	7879035950.6	7,28	26	1139
0.9	19, 22, 12	1, 17, 4, 7, 8	1, 17, 4, 7, 12	8496303481.6	4,80	25	1581
|T| : 4							
0.3	19,22,8, 12	17, 4, 7, 23, 14	17, 4, 7, 12, 14	6442670758.4	24,79	69	1121
0.5	19,22,8, 12	17, 4, 7, 23, 14	17, 4, 7, 12, 14	7428850136.0	17,07	63	1608
0.7	19,22,8, 12	17, 4, 7, 23, 24	17, 4, 7, 24, 12	8222315559.8	11,95	51	1982
0.9	19,22,8, 12	1, 17, 4, 7, 23	1, 17, 4, 7, 12	8652536352.8	6,73	40	1682

Tables 5.2.1 and 5.2.2 indicate that when the interdiction budget increases, optimal cuts and consequently solution time grows. Since higher budget allows NI try different alternatives, the number of iterations increases. Since inner level which is *the p-hub median problem* solution time increases independent from the number of iterations. When the discount factor increases, the number of iterations decreases and the solution time grows.

For the CAB25 network, the most critical hub seems hub facility *12* since it is almost in every interdiction set for different discount factors and interdiction budget. Hub facilities *4, 7, and 17* are selected after most of the interdiction cases. Eventually, they are also optimal facilities without interdiction case. Therefore, these hubs must be in the alternative plan of NU. Also, when the discount factor is smaller, interdiction damage is higher. It makes sense since when the discount factor increases, economies of scale of hub facilities loses economic value.

Table 5.2.2. MCFPIP computations on AP-20 data

Data Set: AP-20
p-hub: 3

α	**Interdicted Hubs**	**Selected Hubs**	**Hubs before interdiction**	**Total Cost-Interdiction**	**Increase (%)**	**Cuts Count**	**Time (s)**
|T| : 1							
0.3	7	18, 20, 10	18, 20, 7	700542	1.24	3	245
0.5	20	19, 7, 24	18, 20, 7	773592	0.82	4	180
0.7	24	20, 7, 14	19, 7, 24	823178	1.07	3	179
|T| : 2							
0.3	20, 7	18, 19, 4	18, 20, 7	708966	2.45	6	186
0.5	20, 7	17, 4, 24	18, 20, 7	784291	2.21	4	198
0.7	7, 24	20, 10, 14	19, 7, 24	837353	1.70	6	301

For the AP20 network, NU’s hub selection is restricted by 3-node since having smaller nodes. Hub facilities *20 and 7* seem most critical hubs of NU. For every alternative setting on interdiction budget and discount factor, NI directly interdicts hub facilities as in facility interdiction problems. However, it is just an issue depending on the example network. For CAB25, interdicted facilities are different from primal optimal hubs as seen in Table 5.2.1.

Next Chapter 6 concludes and discuss the future work of the thesis.

Chapter 6

CONCLUSION AND DISCUSSION

The hub-and-spoke structure firstly emerged in the 1970s in the USA for package delivery and airline networks, and then companies and governments which wanted to expand their supply chains and reduce transportation costs have also considered and implemented the hub-and-spoke structure. Since the hub located networks have an efficient system approach with flexibility and affordability in transport costs and manageability, they have attracted many researchers. The motivation of these researchers has been that finding optimal location of hub facilities in the network is incredibly profitable thanks to the economies of scale principle.

These studies assume that hub networks are run in the perfect environment. However, many natural and intentional disruptions so-called interdiction may threat hub networks. These disruptions may make hubs dysfunctional and cause immense damage to networks since hubs are a critical infrastructure of the networks. Although analysis of these disruptions is of great importance, there are a few studies consider hub interdiction problem. Therefore, we try to contribute the research area with this thesis.

This study examines the discrete interdiction of an uncapacitated hub-and-spoke network. The problem we consider takes the perspectives of two decision makers, e.g. a network user (NU) and a network interdictor (NI). The NU wishes to minimize transportation cost of service flows across the network by facilitating *p* hub facilities, and the NI attempts to worsen the NU's objective by disrupting

NU's facilities. We refer to this problem as the *Multi-Commodity Flow-based p-hub Median Interdiction Problem* (MCFPIP).

MCFPIP incorporates the risks of possible interdiction operations in the initial design of a hub-and-spoke network system by identifying alternative hub location strategies which are both cost-efficient and robust to external disruptions. Unlike previous hub interdiction studies, MCFPIP assumes that NI interdicts most critical facilities of the network and NU can continue its activities on the resulted network after interdiction with *p* hub facilities.

MCFPIP is a two-stage game between two players who have different objectives. Therefore, we model MCFPIP in a bilevel program which is appropriate to model two-stage Stackelberg game. In this interdiction game, NI and NU sequentially make decisions in a noncooperative manner. We adopt a decomposition algorithm based on Benders Decomposition to solve this bilevel program since there is no software on hand for a direct solution of bilevel programs.

Furthermore, we benefit G-MApHMP features to locate hubs of NU in our base model of the p-hub median problem. Therefore, MCFPIP can be applied to incomplete network structures even if they do not have triangle inequality. Hence, it is possible to run it on the cases that would represent network structures in real-world applications.

We focus on deterministic interdiction cases. Stochastic versions of the problem can work on the probabilistic measure for success probability of both NU and NI operations. Instead of discrete interdiction, the continuous case may also be studied. Moreover, protection problem can be added as a third stage to our two-stage interdiction game.

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