



# Variable structure controllers for unstable processes



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## ABSTRACT

A variable structure control (VSC) method for unstable industrial processes is proposed. The proposed control method is able to provide a highly satisfactory system performance and to tackle with robustness issues of the processes in the presence of uncertainties. An ITAE-based numerical tuning algorithm for acquiring optimal control parameters, and a direct auto-tuning mechanism for the proposed controller are also provided. The performance of the proposed VSC method is illustrated on some unstable process models including a continuous stirred tank reactor (CSTR), in order to show its effectiveness, validity and feasibility.

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## 1. Introduction

Process control system designs are mostly based on PID controllers and empirical process models [1–5]. The empirical first-order plus time delay (FOPTD) models can describe dynamics of many processes appropriately for control design aims. Specifically such models are used for tuning PID controllers and stability analysis of the closed-loop processes [1]. The intuitiveness, simplicity and good performance features of the PID (mostly PI) controllers make them the most widely used control strategy today [6–9]. However, PID controllers, like other classical approaches, have robustness vulnerability and may pose performance challenges for unstable and uncertain processes.

In recent years, there has been a great interest in control designs for unstable processes (e.g. unstable FOPTD models) since it is well-known that the performance specifications obtained for a stable model cannot work for an unstable processes. Therefore, many methods have been developed for stabilizing unstable processes including the modified Ziegler–Nichols method [10], mirror mapping [11], truncated predictor feedback control [12], smith predictor based control [13], PID-based controllers [14–18], IMC method [19,20], optimization-based methods [21,22], synthesis method [23], sliding mode control [24,25], and the fuzzy-neural approach [26]. Most of the above methods have additional

adjustable parameters for obtaining controller parameters, complex design procedures, and robustness issues.

The goal of this work is to develop a robust, simple and effective VSC method for unstable processes. Another aim is to provide a direct auto-tuning algorithm for the proposed VSC system without needing a secondary relay method. In the literature, there are very few VSC systems for unstable systems while different switching control strategies similar to gain scheduling approaches can be seen. Most of the given switched-systems have a switching strategy with some PID controllers or continuous change of controller parameters [27]. Some of the studies seen in the literature include: variable structure PID controllers [27–30], a variable parameters based PID controller [31,32], a controller switching between P, PD and PID structures [33,34], and a variable parameters based control [35]. In general, these studies utilize various switching logics to enhance the system performance under operational variations. Some studies have also been considered a specific variable structure system, i.e. sliding mode control methodology [36–39], for the process control systems [24,25,40–43]. In these studies, the integral sliding surface design was used for the reduced-order (FOPDT) models of processes, and some parameter tuning structures similar to empirical PID tuning algorithms were developed for process control systems. However, these methods require the measurement of the derivative of the process output, and thus, can result in poor control performances. For these reasons, this work aims to develop a VSC method having the robustness and good response features of the sliding mode control, and effectiveness and simplicity of PID controllers. Since process control systems often use empirical

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models, i.e. FOPDT models, robustness and setpoint performance of these controllers in the presence of parameter variations and disturbances become important in operations [1,44]. The proposed VSC approach with its direct auto-tuning function can solve all these issues by providing a good setpoint response and robustness.

The organization of the study is as follows: Section 2 introduces the proposed VSC system, some application examples of the controller are given in Section 3, and finally, the conclusion of this work is provided in Section 4.

## 2. The proposed variable structure controller

Some exothermic chemical reactors and biochemical reactors are operated at open-loop unstable steady-states [45]. For open-loop unstable systems, which are difficult to control due to the tight tuning requirements, an appropriate controller must first stabilize the system. In addition, model uncertainty, load disturbance, measurement noise, and set-point response must all be taken into account in a reasonable design method. A drawback of classical (e.g. PID) controllers is that they do not consider all these aspects in a balanced way [1]. More importantly, the robustness of controllers is an unavoidable problem in classical control methods. The robustness problem can be better addressed with a variable structure control system.

An open-loop unstable process can be modeled with [46,47]

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s - 1} e^{-Ls} \quad (1)$$

where  $K$  is the static gain,  $\tau$  is the time constant and  $L$  is the delay. With the use of Taylor series approach, i.e.  $e^{-Ls} \approx 1/(Ls+1)$ , the unstable model (1) can be approximated to

$$\frac{Y(s)}{U(s)} \cong \frac{K}{(\tau s - 1)(Ls + 1)} \quad (2)$$

or in the differential equation form

$$\ddot{y} + \alpha_1 \dot{y} + \alpha_0 y = \beta u \quad (3)$$

where  $\alpha_0 = 1/\tau L$ ,  $\alpha_1 = (\tau - L)\alpha_0$  and  $\beta = K\alpha_0$ . Now, a VSC can be designed to stabilize error dynamics. Due to their inherently robustness against uncertainties, the VSC systems can fit well to such models with a suitable design. In these control methods, a switching surface is usually designed so that the VSC drives the error trajectories of a system onto this surface and keeps these trajectories on the surface for all subsequent times. For the system (3), a switching surface can be designed as

$$\sigma = \dot{e} + \lambda e \quad (4)$$

where  $e = r - y$  with a reference signal  $r(t)$ , and  $\lambda > 0$  is a constant. To bring error trajectories on this surface and keep them there, a suitable VSC can be designed by

$$u = -k_0 y + k_p |\sigma|^{1/2} \text{sat}(\sigma) + u_1 \quad (5)$$

$$\dot{u}_1 = k_i |\sigma|^{1/2} \text{sat}(\sigma)$$

where  $k_0$  and  $k_p$  are proportional gains,  $k_i$  is an integral gain,  $\sigma$  is the switching surface, and  $\text{sat}(\cdot)$  is the saturation function defined by

$$\text{sat}(\sigma) = \begin{cases} \sigma/|\sigma|, & \text{if } |\sigma| \geq 1 \\ \sigma, & \text{if } |\sigma| < 1 \end{cases} \quad (6)$$

In (5), it is assumed that  $|u_1| \leq \mu_3$  for some  $\mu_3 > 0$ . The block diagram of the proposed control system is illustrated in Fig. 1.

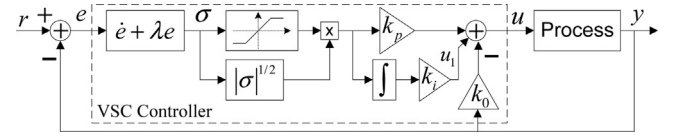


Fig. 1. Block diagram of the proposed VSC system.

By considering the model (3), switching surface (4), and controller (5), a stability analysis of the proposed VSC can be done as follows. First, if a positive definite Lyapunov function is defined by

$$V = \sigma^2/2 + u_1^2/2 \quad (7)$$

then its derivative must be negative definite for stability,

$$\dot{V} = \sigma (\ddot{r} + \lambda \dot{r} + (\alpha_1 - \lambda)\dot{y} + \alpha_0 y - \beta u) - \beta u_1 \sigma + u_1 \dot{u}_1 \quad (8)$$

where it is assumed that  $|\ddot{r}| \leq \mu_2$ ,  $|\dot{r}| \leq \mu_1$ ,  $|y| \leq \mu_0$  for some positive numbers  $\mu_0$ ,  $\mu_1$  and  $\mu_2$ . From (6), it is clear that the controller consists of outer and inner parts. For the outer part of the controller, i.e. for  $|\sigma| \geq 1$ ,

$$\dot{V} = \sigma (\ddot{r} + \lambda \dot{r} + (\alpha_1 - \lambda)\dot{y} + (\beta k_0 - \alpha_0)y - \beta k_p |\sigma|^{1/2} \text{sgn}(\sigma)) + \varphi_0 \quad (9)$$

where the function  $\varphi_0$  can be written as

$$\begin{aligned} \varphi_0 &= -\beta u_1 \sigma + k_i u_1 |\sigma|^{1/2} \text{sgn}(\sigma) \\ &= -\beta u_1 \sigma (1 - |\sigma|^{-1/2}) \\ &= -\varepsilon \beta u_1 \sigma \end{aligned} \quad (10)$$

where  $0 \leq \varepsilon < 1$  and it is assumed that  $k_i = \beta$  for simplicity. Substituting (10) into (9) results in

$$\begin{aligned} \dot{V} &= \sigma (\ddot{r} + \lambda \dot{r} + (\alpha_1 - \lambda)\dot{y} + (\beta k_0 - \alpha_0)y - \beta k_p |\sigma|^{1/2} \text{sgn}(\sigma)) + \varphi_0 \\ &\leq (|\ddot{r}| + \lambda |\dot{r}| + \varepsilon \beta |u_1| - \beta k_p |\sigma|^{1/2}) |\sigma| \\ &\leq (\mu_2 + \lambda \mu_1 + \varepsilon \beta \mu_3 - \beta k_p |\sigma|^{1/2}) |\sigma| \\ &= -(\beta k_p |\sigma|^{1/2} - \hat{\mu}) |\sigma| \end{aligned} \quad (11)$$

where  $\hat{\mu} = \mu_2 + \lambda \mu_1 + \varepsilon \beta \mu_3$ ,  $\lambda \geq \alpha_1$  and  $k_0 = \alpha_0/\beta = 1/K$ . Since  $|\sigma| \geq 1$ , if we choose  $k_p > \hat{\mu}/\beta$ , then  $\dot{V} < 0$ . Namely, whenever  $|\sigma| \geq 1$ ,  $|\sigma(t)|$  will strictly decrease until it reaches the set  $|\sigma| < 1$  in finite time and remains inside the set subsequently. For the inner part of the controller, i.e. inside the set  $|\sigma| < 1$ , the Eq. (8) can similarly be written as

$$\dot{V} = \sigma (\ddot{r} + \lambda \dot{r} + (\alpha_1 - \lambda)\dot{y} + (\beta k_0 - \alpha_0)y - \beta k_p |\sigma|^{1/2} \sigma) + \varphi_1 \quad (12)$$

with

$$\begin{aligned} \varphi_1 &= -\beta u_1 \sigma + k_i u_1 |\sigma|^{1/2} \sigma \\ &= -\beta u_1 \sigma (1 - |\sigma|^{1/2}) \\ &= -\varepsilon_1 \beta u_1 \sigma \end{aligned} \quad (13)$$

where  $0 < \varepsilon_1 < 1$  and again it is assumed that  $k_i = \beta$  for simplicity. Finally,

$$\begin{aligned} \dot{V} &\leq \hat{\mu} |\sigma| - \beta k_p |\sigma|^{5/2} \\ &\leq -(1 - \theta) \beta k_p |\sigma|^{5/2} \end{aligned} \quad (14)$$

where  $0 < \theta < 1$ . The inequality (14) is satisfied for all

$$|\sigma| \geq \left( \frac{\hat{\mu}}{\theta \beta k_p} \right)^{2/3} \quad (15)$$

Hence, the trajectory reaches the ultimate bound set  $\Sigma = \left\{ |\sigma| < \left( \hat{\mu}/(\theta \beta k_p) \right)^{2/3}, |\sigma| < 1 \right\}$  in finite time. This means that the

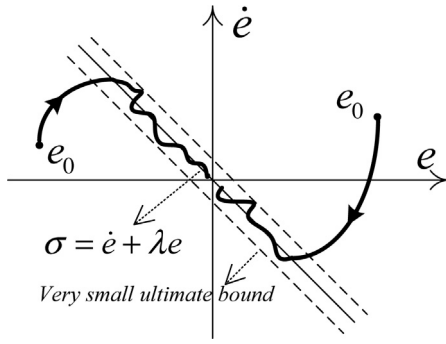


Fig. 2. The phase-portrait of the error dynamics around the switching surface.

tracking error also stays around the origin, but usually not in the origin since  $\sigma = \dot{e} + \lambda e$ . Consequently the practical stability of the proposed controller (5) is guaranteed for the given ultimate bound. This is illustrated in Fig. 2.

It is obvious from stability analysis that there are three conditions to be satisfied: (1)  $k_p > \hat{\mu}/\beta$ , (2)  $\lambda \geq (\tau - t_0)/\tau t_0$  with  $\lambda > 0$ , and (3)  $k_0 = 1/K$ . However, the stability analysis does not provide a straightforward condition for the control term  $k_i$ , and thus it may be selected arbitrarily to get a satisfying steady-state response. A search algorithm for optimal parameter selections is provided in Section 2.3. It should also be noted that if the sign of the model gain  $K$  changes, i.e. if  $K < 0$ , then the sign of the control signal must also be changed ( $u < 0$ ).

### 2.1. Robustness

Robustness of the VSC system under bounded uncertainties must be evaluated since the model (1) and its approximation (2) have inherent uncertainties. The process model (2) with uncertainties can be written as

$$\ddot{y} + \bar{\alpha}_1 \dot{y} + \bar{\alpha}_0 y + d = \bar{\beta} u \quad (16)$$

where  $d$  is a bounded disturbance,  $|d| \leq \mu_4$ ,  $\bar{\alpha}_0 = \alpha_0 + \delta\alpha_0$ ,  $\bar{\alpha}_1 = \alpha_1 + \delta\alpha_1$  and  $\bar{\beta} = \beta + \delta\beta$  with bounded uncertainties  $\delta\alpha_0$ ,  $\delta\alpha_1$  and  $\delta\beta$ . Under these circumstances, the following stability conditions can be obtained via the Lyapunov stability approach as described above,

$$k_p > \frac{\bar{\mu}}{\bar{\beta}}, \quad |\sigma| \geq \left( \frac{\bar{\mu}}{\theta \bar{\beta} k_p} \right)^{2/3} \quad (17)$$

where  $\bar{\mu} \geq \hat{\mu}$  due to the bounds of uncertainties and the disturbance. Consequently, the practical stability of the system is guaranteed under bounded uncertainties.

### 2.2. Various control structures

As in PID control systems, the VSC can be expressed with one-term as

$$u = -k_0 y + k_p |\sigma|^{1/2} \text{sat}(\sigma) \quad (18)$$

and with three-term as

$$u = -k_0 y + k_p |\sigma|^{1/2} \text{sat}(\sigma) + u_1 + u_2$$

$$\dot{u}_1 = k_i |\sigma|^{1/2} \text{sat}(\sigma) \quad (19)$$

$$u_2 = k_d \frac{d}{dt} (|\sigma|^{1/2} \text{sat}(\sigma))$$

Furthermore, other control forms can also be constructed with some combinations of  $u$ ,  $u_1$  and  $u_2$ . The stability of various VSC

structures may also be shown with the similar arguments given above. One interesting point of the VSC is that the integral term of the controller,  $u_1$ , can be omitted for constant reference signals, but it is necessary to have  $u_1$  for time-varying reference profiles to minimize tracking errors.

Finally, the controller (5) exhibits very interesting control features for various exponents of the term  $|\sigma|^p$  for  $0 \leq p \leq 1$ . On the other hand, the proposed controller, i.e. for  $p=0.5$ , utilizes the robustness advantage of the VSC systems and the noise rejection feature of the error-squared control, and thus it is recommended in this work.

### 2.3. Tuning the control parameters

The VSC parameters can be approximately determined from the analytical stability analysis with some trial-and-error, but it is not possible to get optimal parameter settings in such a way. On the other hand, with simulation packages, e.g. Matlab and Maple, numerical simulations can easily be carried out to search for the best possible control parameter settings for the application at hand. In this way, all control design objectives in both the time and frequency domains can be met.

While for some processes, the static gains of the VSC, i.e.  $k_p$ ,  $k_i$  and  $k_d$ , can be calculated from the well-known PID tuning algorithms, a general search algorithm must be developed. The Nelder-Mead simplex algorithm as described in Lagarias et al. [48] can be used for optimizing the VSC system. This algorithm is the basis of Matlab's "fminsearch" optimization algorithm to solve nonlinear optimization problems. In order to get appropriate results, there is a need for performance index. Among the available performance criteria (i.e. IE, IAE, ITAE, ISE, ITSE, ISTE), the ITAE performance index usually gives the most conservative controller settings [49], with the criterion given below,

$$\text{ITAE} = \int_0^{\infty} t |e(t)| dt \quad (20)$$

Now the goal is to find the optimal VSC parameters that minimize the ITAE performance criterion. In the search algorithm, the performance index can be calculated through the Simpson's 1/3 rule [50].

### 2.4. Auto-tuning feature of the variable structure controller

While automatic tuning can be done in many different ways, some commercial controllers use the relay method for auto-tuning in PID controllers. Since the proposed VSC method has a relation with the continuous form of the relay method, it can directly be used for auto-tuning of the control system. To develop such a function, describing functions can be used in analyses. The describing function analysis is an approximate method that can be used to determine whether an oscillation (limit cycle) can occur in a nonlinear feedback system. Fourier series play an important role in determination of the describing function of a nonlinear element since the output of nonlinear element can always be expressed with Fourier series expansion for a given sinusoidal input signal to the nonlinear element [51]. In the describing function analysis, the first harmonics of the Fourier expansion are used by assuming that the first harmonics of the signal coming from the nonlinear element will survive when it passes a low-pass filtering linear system. The describing function is defined as

$$N(a) = \frac{1}{\pi a} \int_0^{2\pi} u_p \sin(\omega t) d\omega t + \frac{j}{\pi a} \int_0^{2\pi} u_p \cos(\omega t) d\omega t \quad (21)$$

where  $N(a)$  is the describing function of a nonlinear element and is only an amplitude,  $a$ , and phase,  $\omega$ , dependent function. The describing function analysis is performed through the Nyquist criterion by defining the following characteristic equation [52]

$$G(j\omega) = -1/N(a) \tag{22}$$

where a limit cycle exists if Eq. (22) has a solution. Similar to the relay based auto-tuning, the one-term VSC structure can be used by setting  $k_i = 0$  in (5), namely,

$$u_p = K_p |\sigma|^{1/2} \text{sat}(\sigma) \tag{23}$$

where the controller normally becomes  $u = -k_0 y + k_p |\sigma|^{1/2} \text{sat}(\sigma)$ , but only the nonlinear part of the controller is considered for the describing function analysis. The fundamental element of (23) can be written in terms of sinusoidal signals for a response to a sinusoidal input,  $\sigma = a \sin(\omega t)$ ,

$$u_p \cong \begin{cases} k_p \sqrt{a} \sin(\omega t), & \text{if } a > 1 \\ k_p a \sqrt{a} \sin(\omega t), & \text{if } a \leq 1 \end{cases} \tag{24}$$

Substituting (24) into (21), a describing function can be found as

$$N(a) = \begin{cases} k_p / \sqrt{a}, & \text{if } a > 1 \\ k_p \sqrt{a}, & \text{if } a \leq 1 \end{cases} \tag{25}$$

Now the function (25) can be used to tune VSC parameters automatically. Let the ultimate gain be  $K_u = N(a)$  and the ultimate period be  $P_u = 2\pi/T$  for the signal period  $T$ . Then, the VSC parameters can be defined as

$$k_p = K_u / 2.2, \quad k_i = 1.2 k_p / P_u \tag{26}$$

Eq. (26) may not give optimal results as described in Section 2.3, but can provide satisfactory results for some unstable systems. Fig. 3 shows the block diagram for the describing function analysis of the VSC system. Since  $u_p$  can be approximated to sinusoidal signals as in (24), i.e. describing function calculations are almost exact and much closer to the conventional sine-wave test, the proposed VSC method is better than the relay and saturation feedback based tests. The other advantages of the above auto-tuning procedure include: avoiding trial-and-error search, not operating near the instability limit, easy to automate, and avoiding an additional relay system.

### 3. Applications

In the following examples and a CSTR application, it will be shown that the VSC method can significantly improve the system performance and robustness. The tuning approach will be based on the nonlinear optimization method described in Section 2.3.

#### 3.1. Example 1: an unstable process

Consider an unstable system described by

$$G(s) = e^{-0.25s} / (s - 1) \tag{27}$$

The VSC and PI controller parameters for the reduced order model (27) are obtained via the optimal search algorithm described

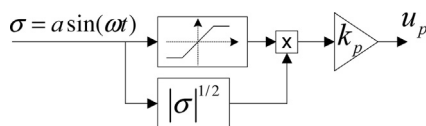


Fig. 3. Block diagram for describing function analysis.

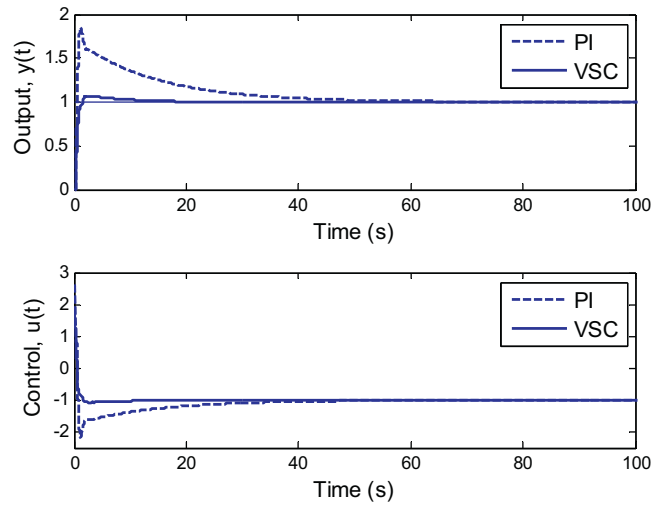


Fig. 4. Time responses of the VSC and PI controller for the given unstable system.

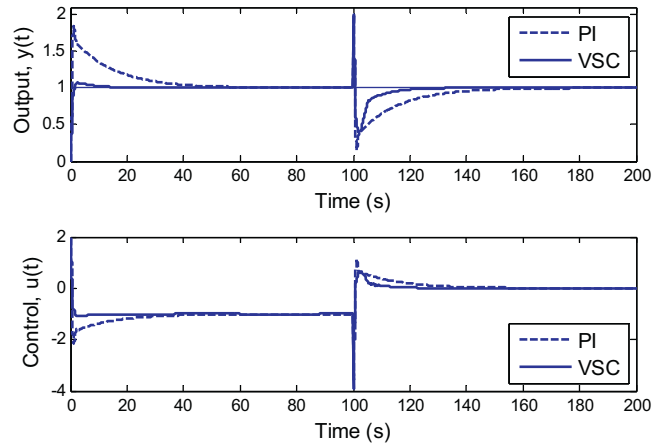


Fig. 5. Disturbance rejection response of the VSC and PI controller.

in Section 2.3. The VSC control parameters are found as  $k_p = 0.5$ ,  $k_i = 0.1$  and  $\lambda = 5$  (for PI controller,  $k_p = 2.55$  and  $k_i = 0.1$ ). Fig. 4 illustrates the setpoint responses of the VSC and PI controller for the system (27). It is clear that the VSC provides much faster response and smaller overshoot compared to the PI controller.

The load disturbance rejection performance of the VSC system is shown in Fig. 5. A step load disturbance is added to system at  $t = 100$  s, and one can observe that the VSC provides better disturbance rejection response.

Performances of the controllers under model uncertainties are shown in Fig. 6. The model uncertainties are considered to be order increase in the process models, e.g.  $G(s)G_a(s) = G(s)/(\tau_a s + 1)$  with a time constant  $\tau_a > 0$ . When the order of the model dynamics increases with different time constants (i.e.  $\tau_a = 0.1$  and  $\tau_a = 0.36$ ), it is seen from figures that the VSC system gives more conservative results compared to the PI controller. When the time constant of the unmodeled dynamics increases from  $\tau_a = 0.1$  to  $\tau_a = 0.36$  as in Fig. 6(b), while the PI controller results in an unstable system response, the VSC stabilizes the process in a short time. Hence, this shows that the VSC system increases robustness of the closed-loop system.

#### 3.2. Example 2: a non-minimum phase unstable process

The control of unstable systems with minimum/non-minimum phase processes is a challenging one [13]. Since they are easy

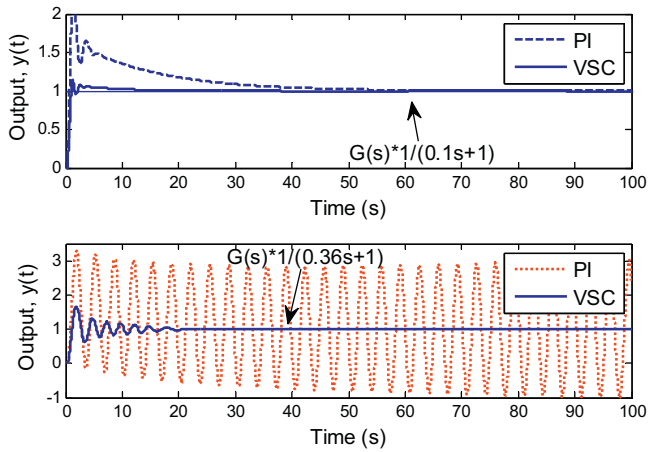


Fig. 6. Controller performance under model uncertainties.

to design and tune, simple controllers are very valuable for the process industry. Consider an unstable non-minimum phase delayed system

$$G(s) = e^{-0.25s}(1 - 0.5s)/(s - 1) \quad (28)$$

The delayed system has an unstable pole and a positive zero. Appropriate VSC gains for the system are  $k_p = 0.032$ ,  $k_i = 0.005$  and  $\lambda = 3$  (for PI controller,  $k_p = 1.26$  and  $k_i = 0.02$ ). As seen in Fig. 7, both VSC and PI controller have some overshoot, but the VSC provides faster transient response and much better load disturbance response, as well as more conservative control requirements. Specifically, the overshoot is significantly decreased with the VSC.

### 3.3. An application: the continuous stirred tank reactor

Consider a well-mixed CSTR problem described by [53–55]

$$\begin{aligned} \dot{x}_1 &= -(1 + \delta_1)x_1 + 2\delta_2x_{2d}x_2 + \delta_2x_2^2 \\ \dot{x}_2 &= \delta_1x_1 - (1 + \delta_2x_{2d} + 2\delta_3x_{2d})x_2 - (\delta_2 + \delta_3)x_2^2 + u \\ \dot{x}_3 &= 2\delta_3x_{2d}x_2 - x_3 + \delta_3x_2^2 \end{aligned} \quad (29)$$

where  $x_1$ ,  $x_2$  and  $x_3$  are the dimensionless concentrations of the reactants in the reactor zone, the measurable reactant concentration in the exit stream is  $x_2$ , and the manipulated variable  $u$  is the dimensionless feed concentration. The system parameters are given by  $\delta_1 = 3$ ,  $\delta_2 = 0.5$ ,  $\delta_3 = 1$ , and  $x_{2d} = 0.8796$ . The CSTR model

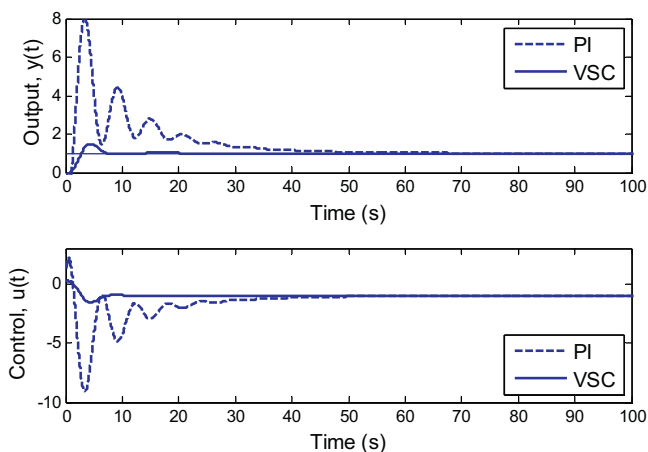


Fig. 7. Responses to set-points and load disturbances of the oscillatory process with VSC and PI controller.

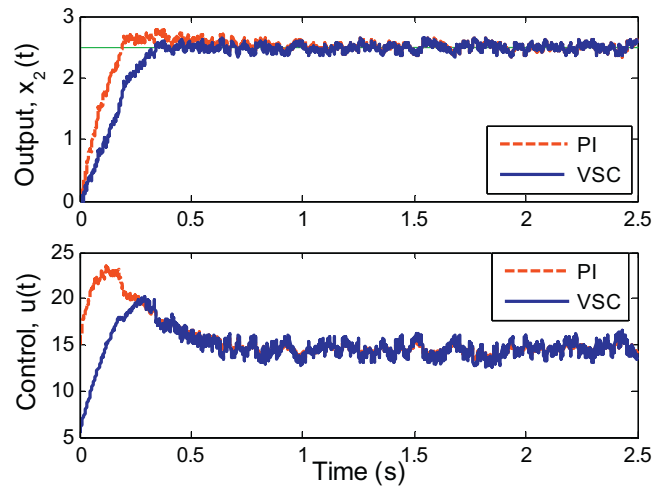


Fig. 8. Responses to set-point of the CSTR under noise with VSC and PI controllers.

given in (29) is highly nonlinear and have two equilibrium points (one of them is unstable) for the given parameter values. The control aim is to keep the desired concentration,  $x_2$ , as close as possible to its set point.

For the given system, suitable VSC gains can be found from the nonlinear optimization method (see Section 2.3) as  $k_p = 1$ ,  $k_i = 20$  and  $\lambda = 10$ , and the PI gains are obtained as  $k_p = 6$  and  $k_i = 103$  using the MATLAB’s tuning algorithm. The measurement noise is assumed to be normally distributed with the magnitude around  $\pm 10\%$  of the set-point. Due to the measurement noise, to limit the high frequency gain of the derivative term in (4), a derivative filter is used, i.e.  $\dot{e} = e[s/(0.5s + 1)]$ . Fig. 8 shows set-point responses of the VSC and PI controlled CSTR under noisy measurements. It is seen that the VSC significantly improves the transient response of the system (non-overshoot response) while requiring smaller control energy compared to the PI controller. The effects of noise on the system measurement and control signal are similar for both methods.

## 4. Conclusion

A variable structure control approach is proposed for unstable processes for getting a highly satisfactory control performance and robustness against uncertainties and disturbances. The VSC system is designed in the form of the PID controllers so that it can be directly applied to existing automatic control software and hardware with some software modifications. It is shown that: (1) the VSC method is robust under bounded uncertainties, (2) changes in the process parameters can result in instability with PID controllers, but stable limit cycle with the VSC system, (3) a highly satisfactory system performance is obtained with conservative control requirements, (4) the VSC method can be directly used to acquire an auto-tuning function similar to relay method without a secondary relay system. Consequently, in order to enhance control performances and controller robustness, the proposed VSC method can directly be used for controlling processes instead of PID controllers.

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