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A variant SDDP approach for periodic-review approximately optimal pricing of a slow-moving a item in a duopoly under price protection with end-of-life return and retail fixed markdown policy

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A R T I C L E I N F O

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A B S T R A C T

In this paper, we examine a selling environment where a manufacturer-controlled retailer and an independent retailer sell a slow-moving *A* item. The manufacturer offers the independent retailer a price protection contract stipulating that the manufacturer reimburses the independent retailer in case of a reduction in the wholesale price. The price set by the independent retailer is assumed to be determined by Retail Fixed Markdown (RFM) policy. The manufacturer also offers the independent retailer a special discount rate for the replenishment orders and the retailers are assumed to follow *(R, S)* inventory replenishment policy. The manufacturer adopts a periodic-review pricing strategy and the mean demand observed by each retailer in a given period depends on the prices. We also take the customers choosing no-purchase option into account. We employ multinomial logit (MNL) models to forecast customers' preferences based on retail prices. The retailers' market shares are estimated by customized choice probability functions. We propose stochastic programming models to determine the manufacturer's pricing strategy. Then, we propose a variant Stochastic Dual Dynamic Programming (SDDP) algorithm to determine the manufacturer's approximately optimal pricing strategy by getting around three curses of dimensionality. Then, we move on to the observations on the impact of four critically important contractual parameters on the price, the market shares and the expected total net profits and finally discuss some possible approaches for the selection of the best compromise values of those contractual parameters.

1. Introduction

In high-tech industry, customers tend to purchase technologically advanced brand new products or the improved models of the products they already have. Lee, Padmanabhan, Taylor and Wang (2000) state that in the personal computer industry, products face rapid obsolescence that gives rise to slumps in prices throughout their life cycles so sellers are confronted with high demand uncertainty. This tendency compels manufacturers to make some changes in their product mixes. With the development, production and introduction of some brand new products, the old products are offered at discounted prices to the customers that have relatively low budgets. As manufacturers can sell their products to the end customer via its own retailers, they may also prefer collaborating with some retailers in order to reach much more customers in the market. For this purpose, manufacturers should offer some privileges to entice retailers to keep the inventory of its products. Especially, external retailers want to be protected against sudden drops in the wholesale prices at which they purchase the products. Sourirajan, Kapuscinski and Ettl (2008) state that price protection is intended to induce distributors and retailers to keep adequate inventory by protecting them against sudden drops in the price of the corresponding product. Manufacturers offer a price protection contract by which they assure retailers that they are committed to reimbursing retailers the amount of reduction in wholesale prices per product for the inventory retailers have in stock.

In presence of such restrictions, manufacturers might offer some other privileges than price protection. One of these privileges is mid-life returns. If a manufacturer grants a retailer the opportunity of returning some of its inventory at any time of the selling horizon, then it also accepts refunding the retailer some money per returned product. Likewise, if the product in question will be withdrawn from the market after

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a selling horizon of a predetermined length, retailers can be allowed to return their remaining on-hand inventory at the end of that selling horizon in exchange for some refund. This is called end-of-life returns.

In the literature, there is a variety of research papers where the effects of different price commitment policies on channel coordination in different selling environments are analyzed in a two-period case where there exist one manufacturer and one retailer (Taylor, 2001; Zhang, 2008; Lu, Song, & Regan, 2007; Chen & Xiao, 2011; Lee et al, 2000). Researchers present the conditions that have to be satisfied to ensure channel coordination for some policies which are capable of coordinating a supply chain. These policies are price protection, mid-life returns and end-of-life returns. There also exist research papers in which some supplementary policies are evaluated in conjunction with price protection, mid-life returns and end-of-life return in a two period case where there exist one manufacturer and one retailer (Lee & Rhee, 2007; Wang, 2002).

These studies mostly focus on which inventory replenishment or return policy is optimal and researchers aim to determine the optimal policy parameters. In this kind of research papers, the retail price and the wholesale price set in each period of the selling horizon are assumed to be fixed and researchers employ demand distributions that are estimated based on fixed retail prices. However, it is also intriguing whether or not the evaluated price commitment policies and return policies are capable of coordinating a supply chain and providing a win–win outcome in an environment where retail prices are decision variables and they have an influence on demand distributions. For that purpose, we only focus on pricing strategies in this study by excluding channel coordination and win–win outcome concerns and reckoning with price-dependent consumer behaviors in order to lay a foundation for further researches on channel coordination in selling environments where responsive pricing strategy is adopted instead of pre-announced pricing strategy.

In the literature, there are research papers in which pricing and inventory control decisions are studied and analyzed simultaneously. Chen, Chen, Keblis and Lee (2019) study a selling environment where a deteriorating product that is assumed to have a short lifecycle is sold throughout a finite selling horizon of multiple periods. They assume deterministic, stock level-dependent, time-varying and price-dependent demand for the product and develop an algorithm to determine a profit-maximizing replenishment and pricing policy. Ghoreishi, Mirzazadeh, Weber and Nakhai-Kamalabadi (2015) build an Economic Order Quantity model for non-instantaneous deteriorating items for which inflation- and selling price-dependent demand is observed by allowing partial backlogging and customer returns. They propose an efficient algorithm intended to simultaneously optimize the selling price, the length of the replenishment cycle and the length of time when shortage does not occur. Mishra (2017) builds a model for a deteriorating item for which stock level- and selling price-dependent demand is observed by assuming Weibull deterioration and partial backlogging. The author proposes a simple algorithm designed to simultaneously optimize the selling price, the replenishment schedule and the order quantity with the purpose of maximizing the total profit. Nagaraju, Rao and Narayanan (2016) focus on both a centralized and a decentralized three-echelon inventory system consisting of a manufacturer, a distributor and a retailer. The author assumes that selling price-dependent deterministic demand is observed for the product and the selling price is a function of the replenishment quantity with dependence factor. Some managerial insights on the optimal selection of replenishment quantity and shipment frequency are set forth in the paper. In another research paper, pricing and inventory decisions for a two-echelon inventory system are discussed by assuming nonlinear price-dependent demand (Nagaraju, Kumar, & Narayanan, 2018). Agi and Soni (2020) build a deterministic model and propose an algorithm meant for the simultaneous optimization of the selling price, the cycle length, the order quantity and the end-of-cycle inventory for a perishable product. The authors assume that demand depends on the selling price, the current inventory level and the freshness condition.

If a manufacturer and the retailers with which it collaborates sell a product to the end customer, then there exists a natural competition. Therefore, unlike the previous studies, the impact of the retail prices on consumers' purchasing behavior has to be taken into account, as well. That is, it is not convenient to employ even price-dependent demand functions. Instead, price-dependent stochastic demand distributions have to be employed. As a starting point for such analyses, we commit ourselves to examining the impact of price protection and end-of-life return opportunity on the optimal retail price and the actors' profits. However, we do not deal with the determination of the optimal inventory replenishment policy and the optimal return policy.

In this study, a manufacturer-controlled retailer and an independent retailer sell a slow-moving *A* item throughout a finite selling horizon which is partitioned into periods. The manufacturer sets the retail price at the beginning of each period and the manufacturer-controlled retailer sells the product to the end customer at that price. The independent retailer offers the end customer a discounted price in accordance with Retail Fixed Markdown (RFM) policy. We assume a non-increasing price environment resulting from the depreciation of the product over time. The manufacturer offers the independent retailer price protection meant for the protection against slumps in the retail price. The manufacturer also allows the independent retailer to return the entire unsold inventory at the end of the selling horizon. Both retailers are allowed to place a replenishment order at the beginning of each period. In this study, we make an assumption on the inventory replenishment policies that the retailers follow throughout the selling horizon so that we are not concerned with the optimization of replenishment policies. We also employ price-dependent stochastic demand distributions by taking the influence of the retail prices on consumers' valuations about the retailers into consideration.

The manufacturer aims to develop a profit-maximizing pricing strategy. The optimal pricing strategy can be determined by modeling the problem through dynamic programming approach. However, it is impossible to determine the optimal retail price in a given period for each possible state because the optimal retail price in a given period of the selling horizon depends on the retail price set in the previous period and the retail price is a continuous decision variable. That is, the state space is infinite so the model is plagued by one of the three curses of dimensionality defined in Powell (2007). Furthermore, the decision space and the random event space are also infinite so all the three curses of dimensionality trouble the determination of the optimal pricing strategy. Therefore, our objective is to propose a suitable method meant to determine an approximately optimal pricing strategy for the manufacturer and to analyze the impact of price protection and end-of-life return opportunity on this approximately optimal pricing strategy and the manufacturer's and the independent retailer's true expected total profits given that strategy.

In the literature, some approximate dynamic programming algorithms are proposed to circumvent the difficulties that arise from the existence of three curses of dimensionality (Powell, 2007). The fast convergence of these algorithms necessitate decent estimations of the value functions such as pre-decision profit-to-go functions, post-decision profit-to-go functions, Q-factors etc. However, the existence of a proper and reliable way of estimating the value functions is questionable. Furthermore, the optimization is done sequentially by starting from the first stage and generating a realization from the distribution of random event occurring between stages on each iteration. That is, a single state is visited at each stage on each iteration. Even if the demand distributions depend on the decision variable in our problem, sequential optimization helps us calculate the retailers' market shares given the optimal action through special market share functions and estimate the demand distributions based on those market shares at each stage. For that reason, these algorithms are adaptable to our problem. However, the applicability is also a very critical evaluation measure in the selection of a suitable adaptable methodology.

These algorithms entail updating the estimations of the value functions through a smoothing operation on each iteration by using the estimations of the previous iteration and the approximate optimal values of the current iteration. However, the concern about the smoothing operation is the selection of an appropriate step size. Moreover, these algorithms terminate when the number of iterations reaches the preset maximum value. However, the selection of that value is a little bit problematic in our case because the convergence is bound to require a very large number of iterations. Therefore, these algorithms are not applicable to our problem although they are adaptable because of the complicated structure of the problem.

Unlike the approximate dynamic programming algorithms, Stochastic Dual Dynamic Programming (SDDP) algorithm first proposed by Pereira and Pinto (1991) and analyzed further in Chen and Powell (1999), Donohue and Birge (2006), Linowsky and Philpott (2005) and Philpott and Guan (2008) for finite random data process skillfully deals with the estimation of the post-decision profit-to-go functions by deriving an upper bound on each iteration and it has a legitimate stopping criterion. However, the random data process is assumed to be finite in these papers. As in our case, the random data process is generally infinite in most real-life applications. For this reason, Shapiro (2011) discusses how the SDDP algorithm can be implemented when the random data process is infinite. Since the algorithm is also well-suited to the cases in which random data process is infinite, this algorithm is more applicable to our case compared to the approximate dynamic programming algorithms. For that reason, we model our problem through a

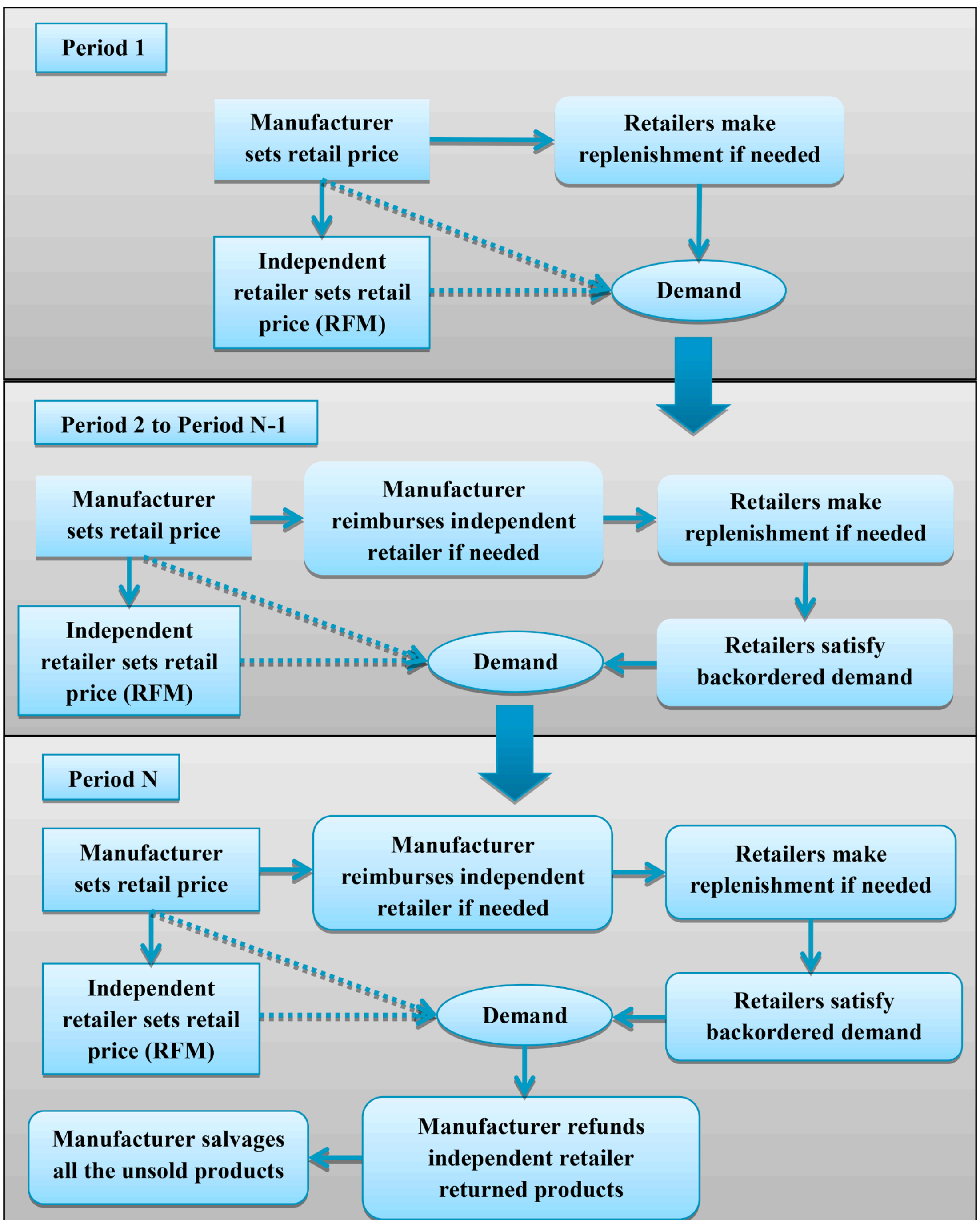

Fig. 2.1. . The sequence and interactions of decisions, actions and events in a selling horizon of *N* periods.

stochastic programming approach.

SDDP algorithm is a simulation-oriented method the initialization of which necessitates the generation of a number of realizations from random event distributions. However, demand distributions that depend on the decision variable preclude the direct generation of demand realizations in our case. For this reason, we propose a modified version of the SDDP algorithm to determine the manufacturer's approximately optimal pricing strategy. By doing so, we also want to shed light on how the problems afflicted by three curses of dimensionality in which random event distribution depends on the decision variable can be dealt with in case the parametric expression of the optimal solution is not possible.

The organization of the research paper is as follows; in the next section, we provide the full problem definition by defining the boundaries of the research built on some assumptions. In the third section, we present the stochastic programming model to be solved to determine the manufacturer's optimal pricing strategy. In the fourth section, we explain the variant SDDP algorithm we propose to circumvent the three curses of dimensionality and to determine the manufacturer's approximately optimal pricing strategy. In the fifth section, we present the results of the numerical experiments carried out to observe how the changes in the four contractual parameters playing a significant role in the manufacturer's pricing decisions impact the approximate optimal price, the retailers' market shares and their true expected total net profits. In the sixth section, we summarize the study and share the conclusions we draw.

2. Problem definition

In this problem, a manufacturer-controlled retailer and an independent retailer sell a slow-moving A item to the end customer throughout a finite selling horizon. The manufacturer is responsible for the production and the delivery of the corresponding product to the retailers. At the beginning of each period, the manufacturer determines the retail price at which the manufacturer-controlled retailer will sell the product to the end customer as shown in Figure 2.1. A non-increasing price environment is assumed by neglecting some extraordinary external factors such as erratically changing foreign currency parities, inflation rate, interest rate etc. that might have an impact on the manufacturing cost of the product. That is, the product keeps depreciating over time so the manufacturer does not increase the retail price throughout the selling horizon. In purpose for enticing the independent retailer into keeping the inventory of the product during the selling horizon, the manufacturer offers the independent retailer price protection in case of a reduction in the retail price. The manufacturer is committed to reimbursing the independent retailer for a fixed proportion of the independent retailer's on-hand inventory whenever it decreases the retail price. From the second period on, reimbursement should be fulfilled at the beginning of each period in which the manufacturer decides to sell the product at a lower price as shown in Figure 2.1 provided below.

Both of the retailers are allowed to make replenishment at the beginning of each period after the manufacturer sets the retail price as shown in Figure 2.1. The necessary lead time for the production and the delivery of replenishment orders is assumed to be negligibly short. Since the purpose of this study is to determine the manufacturer's optimal pricing strategy instead of optimal replenishment policies, an assumption is made about the replenishment policies that these retailers follow throughout the selling horizon. This assumption is inspired by the research papers that study channel-coordinating replenishment policies in case of fixed retail and wholesale prices. By the assumption, both of the retailers follow order-up-to inventory replenishment policy (R, S) which is proven to coordinate supply chains in many problem settings discussed in Lee et al. (2000), Lee and Rhee (2007) and Liu, Fry, Qin and Raturi (2012). The independent retailer's order-up-to level is negotiable because retailers are inclined to order in large batches in presence of price protection and end-of-life return opportunity. Replenishment policy parameters are fixed in the price commitment contract and they cannot be changed during the selling horizon. It is assumed that both retailers have no stock before they make replenishment at the beginning of the selling horizon.

The manufacturer enables the independent retailer to purchase products at a discounted price. After the manufacturer sets the retail price in a given period, the wholesale price is determined by discounting the retail price. The discount rate is assumed to be fixed over time. Both of the retailers are allowed to backorder a fixed maximum allowable amount of demand in each period except the last one in case they observe excess demand. The retailers determine the size of their replenishment orders by reckoning with backordered demand. From the second period on, the backordered demand is satisfied after the replenishment order is delivered at the beginning of each period as shown in Figure 2.1. The retailers offer their customers a special discount for the backordered demand. That discount is applied on the price that customers would have paid if the product had been available in the previous period. No lost sales cost is incurred for the other customers turned down in a stockout.

After the manufacturer sets the retail price in a given period, the retail price at which the independent retailer will sell the product to the end customer is determined by Retail Fixed Markdown (RFM) policy as shown in Figure 2.1. That price is computed by marking down the retail price set by the manufacturer by a fixed rate. The objective with the application of RFM policy is to lure customers from a lower-income segment so as to raise revenue. The markdown rate has to be less than the discount rate for the independent retailer's profitability throughout the selling horizon.

Since these retailers are natural competitors in the market, the retail prices have an influence on their demand distributions as shown by dashed lines in Figure 2.1. Poisson distribution is a perfect fit to model demand behavior for a slow-moving A item as proposed by Silver, Pyke and Peterson (1998). Therefore, the distribution of the number of potential customers in each period is assumed to be Poisson with an estimated and known mean value. Potential customers either purchase the product from either retailer or leave the market. The retailers' market shares given the retail prices can be estimated by employing a suitable choice model. The demand observed by each retailer in a given period is also Poisson distributed since a Poisson process can decompose into Poisson sub-processes. The mean demand observed by a given retailer in a given period equals the mean number of potential customers multiplied by the retailer's market share.

The independent retailer is allowed to salvage its entire remaining inventory by returning the unsold products to the manufacturer at the end of the selling horizon in return for a refund. The manufacturer also salvages its leftover inventory and the products returned by the independent retailer.

We model this problem through stochastic programming approach to maximize the manufacturer's expected total profit. The stochastic programming model is presented in the following section.

3. Mathematical models

In this section, the stochastic programming model constructed to maximize the manufacturer's expected total profit is presented. The length of the selling horizon is assumed to be N periods. The notation used throughout the section is shown in Table 3.1 presented below.

In the first period, the manufacturer sets the retail price and then the manufacturer-controlled retailer and the independent retailer make replenishment to raise their inventory levels to their order-up-to levels S_1^m and S_1^r, respectively. As a result, the manufacturer makes some post-decision profit by selling some products to the independent retailer. After replenishments, the manufacturer generates some extra post-decision revenue by selling the product to the end customer in the first period and selling some products to the independent retailer in case of a replenishment order at the beginning of the second period. There-

Table 3.1
Notation.

r	Inventory holding cost per dollar per period
θ	Discount rate offered to customers for backordered demand
β	Discount rate offered to the independent retailer for replenishment orders
α	Reimbursement rate
c_t	Production cost per product in period t $\forall t \in \{1,2,\ldots N\}$
D_t^m	Demand observed by the manufacturer-controlled retailer (m) in period t $\forall t \in \{1,2,\cdots N\}$
D_t^r	Demand observed by the independent retailer (r) in period t $\forall t \in \{1,2,\cdots N\}$
D_t	Ordered pair (D_t^m, D_t^r) of demands observed by retailers in period t $\forall t \in \{1,2,\cdots N\}$
N_t^m	Maximum allowable amount of demand that can be backordered by manufacturer-controlled retailer (m) in period t $\forall t \in \{1,2,\cdots N\}$
N_t^r	Maximum allowable amount of demand that can be backordered by independent retailer (r) in period t $\forall t \in \{1,2,\cdots N\}$
S_t^m	Order-up-to level that the manufacturer-controlled retailer (m) needs to place a replenishment order if inventory level goes below at the beginning of period t $\forall t \in \{1,2,\cdots N\}$
S_t^r	Order-up-to level that the independent retailer (r) needs to place a replenishment order if inventory level goes below at the beginning of period t $\forall t \in \{1,2,\cdots N\}$
I_t^m	Manufacturer-controlled retailer (m)'s on-hand stock right before observing demand in period t $\forall t \in \{1,2,\cdots N\}$
I_t^r	Independent retailer (r)'s on-hand stock right before observing demand in period t $\forall t \in \{1,2,\cdots N\}$
w_m	Salvage value per product of manufacturer-controlled retailer (m)'s unsold inventory at the end of the selling horizon
w_r	Refund per product returned by the independent retailer (r) at the end of the selling horizon
p_t	Retail price set by the manufacturer in period t $\forall t \in \{1,2,\cdots N\}$

fore, the retailers' inventory levels after replenishments and the demand observed by each retailer in the first period are determinants of the manufacturer's pricing decision in the second period. For that reason, a post-decision profit-to-go function ($\varphi_1(S_1^m, S_1^r, p_1)$) of the retailers' inventory levels (S_1^m and S_1^r) after replenishments and the price (p_1) set by the manufacturer in the first period connects the first-stage problem to the second-stage problem. The post-decision profit-to-go function of a given period returns the optimal expected total profit the manufacturer makes from after it sets the retail price and the retailers make replenishments till the end of the selling horizon given the inventory levels after replenishments and the retail price set for the given period. The post-decision profit-to-go function of the first-stage problem is exactly the expected value of the pre-decision profit-to-go function ($Q_2(S_1^m, S_1^r, p_1, D_1)$) of the second-stage problem over the ordered pair (D_1) of demands observed by the retailers in the first period. The model that has to be solved in the first period is as follows:

$$\max_{p_1 \in A_1} - S_1^m . c_1 + ((1-\beta).p_1 - c_1).S_1^r + \varphi_1(S_1^m, S_1^r, p_1), \tag{3.1}$$

where

$$A_1 = \{p_1 \in \mathbb{R} : p_1 \geq 0\}, \tag{3.2}$$

$$\varphi_1(S_1^m, S_1^r, p_1) = \mathbb{E}_{D_1}\left[Q_2(S_1^m, S_1^r, p_1, D_1)\right]. \tag{3.3}$$

At the beginning of a given intermediate period *t*, the manufacturer incurs inventory holding cost per product carried over from period *t-1*, earns some money by selling products to the independent retailer in case the independent retailer places a replenishment order, and incurs some production cost stemming from the retailers' replenishment orders if there is any. The retailers' replenishment orders also cover the products backordered in the period *t-1*. Since there is a non-increasing price environment, if the manufacturer changes the price of the product, it reimburses the independent retailer for a fixed proportion of the unsold inventory carried over by the independent retailer from period *t-1* in compliance with the price commitment contract. Therefore, there exists a reimbursement cost term in the objective function. The objective function also contains the earnings from selling products in period *t-1*. Furthermore, we know that the retailers' inventory levels at the beginning of the period have an influence on the manufacturer's pricing decision. For that reason, we have to solve the model of period *t* for the retailers' all possible inventory levels before they observe demand in period *t-1* and all possible amounts of demand that they can observe in period *t-1*. The possible inventory levels that a given retailer can have at the beginning of period *t-1* range between the given retailer's order-up-to level in period *t-1* and the maximum of its order-up-to levels till period *t-1*. Then, the model that has to be solved for a given intermediate period *t* ($t \neq 1$) is as follows:

$$\begin{aligned}
Q_t(I_{t-1}^m, I_{t-1}^r, p_{t-1}, D_{t-1}) = \max_{p_t \in A_t} & - max\{min\{D_{t-1}^m - I_{t-1}^m, N_{t-1}^m\}, 0\}.\theta.p_{t-1} \\
& + min\{D_{t-1}^m, I_{t-1}^m + N_{t-1}^m\}.p_{t-1} \\
& - max\{I_{t-1}^m - D_{t-1}^m, 0\}.c_{t-1}.r \\
& - max\{S_t^m - I_{t-1}^m + D_{t-1}^m, 0\}.c_t \\
& + max\{D_{t-1}^m - I_{t-1}^m - N_{t-1}^m, 0\}.c_t \\
& + max\{S_t^r - I_{t-1}^r + D_{t-1}^r, 0\}.((1-\beta).p_t - c_t) \\
& - max\{D_{t-1}^r - I_{t-1}^r - N_{t-1}^r, 0\}.((1-\beta).p_t - c_t) \\
& - max\{I_{t-1}^r - D_{t-1}^r, 0\}.\alpha.(p_{t-1} - p_t) \\
& + \varphi_t(IL_t^m(I_{t-1}^m, D_{t-1}^m), IL_t^r(I_{t-1}^r, D_{t-1}^r), p_t)
\end{aligned} \tag{3.4}$$

where

$$A_t = \{p_t \in \mathbb{R} : p_t \leq p_{t-1}, p_t \geq 0\}, \tag{3.5}$$

$$\begin{aligned}\varphi_t(IL_t^m(I_{t-1}^m, D_{t-1}^m), IL_t^r(I_{t-1}^r, D_{t-1}^r), p_t) = \mathbb{E}_{D_t}\big[Q_{t+1}(IL_t^m(I_{t-1}^m, D_{t-1}^m),\\ IL_t^r(I_{t-1}^r, D_{t-1}^r), p_t, D_t)\big],\end{aligned} \tag{3.6}$$

$$IL_t^m(I_{t-1}^m, D_{t-1}^m) = max\{S_t^m, I_{t-1}^m - D_{t-1}^m\}, \tag{3.7}$$

$$IL_t^r(I_{t-1}^r, D_{t-1}^r) = max\{S_t^r, I_{t-1}^r - D_{t-1}^r\}. \tag{3.8}$$

The functions shown in Equation (3.7) and Equation (3.8) return the manufacturer-controlled retailer's and the independent retailer's post-replenishment inventory levels at the beginning of the period. At the end of the selling horizon, the independent retailer returns its unsold inventory to the manufacturer and gets refunded. The manufacturer salvages the manufacturer-controlled retailer's unsold inventory and the products returned by the independent retailer. Then, the profit function of the dummy period *N + 1* given the retailers' inventory levels before observing demand in period *N*, demand observed by the retailers in period *N* and the price set by the manufacturer in period *N* is as follows:

$$\begin{aligned}Q_{N+1}(I_N^m, I_N^r, p_N, D_N) = & -max\{I_N^m - D_N^m, 0\}.c_N.r + D_N^m.p_N - max\{D_N^m - I_N^m, 0\}.p_N \\
& + max\{I_N^m - D_N^m, 0\}.w_m + max\{I_N^r - D_N^r, 0\}.(w_m - w_r).\end{aligned} \tag{3.9}$$

In this model, the state space, the decision space and the random event space are infinite. For that reason, only if closed-form expressions can be obtained for the post-decision profit-to-go functions, the exact optimal price that the manufacturer should set to maximize its expected total profit can be determined. However, it is not possible to solve the optimization problems parametrically. For that reason, we propose a variant SDDP algorithm that can be implemented to determine the approximately optimal price in the following section.

4. Methodology

The multi-stage decision-making is a very troublesome task if a sort of randomness should be taken into account and the random event is identified by a continuous random variable which can inherently take on infinitely many values. However, even the discreteness of the randomness does not facilitate the process if the support of the corresponding discrete random variable is infinite. The hardness triggered by the infinite support of a random event arises from infinite random event space which is one of the curses of dimensionality elaborated on by Powell (2007).

Furthermore, even if the random variable characterizing a random event takes on finitely many values, the existence of an infinite decision space most of the time renders it impossible to keep track of the evolution of optimal actions over time. In that case, the attainment of the exact optimal action at each stage necessitates deriving the closed-form expression of the profit-to-go function of the following stage which provides the expected total profit made till the end of the horizon given the action at that specific time epoch. Under these circumstances, if there is no effective means of circumventing this trouble, the strategist is compelled to relinquish exact optimal actions and driven to seek out a way to obtain an approximate solution. In our model, we are up against such a trouble, as well. In compliance with the problem definition, the manufacturer is allowed to set a continuous price and the distribution of the random demand observed by the manufacturer-controlled retailer and the independent retailer shows a Poisson behavior. For this reason, we have to look for a methodology that supplies us with an implementable set of procedures leading to an approximate optimal pricing policy.

Pereira and Pinto (1991) proposes such an algorithm called Stochastic Dual Dynamic Programming (SDDP) specialized in multi-stage decision-making. In various research papers such as Chen and Powell (1999), Donohue and Birge (2006), Linowsky and Philpott (2005) and Philpott and Guan (2008), this algorithm is extended and analyzed for the case where random data process is finite. However, the random data process is infinite in our study. As an extension to the earlier research, Shapiro (2011) discusses the implementation of the SDDP algorithm in case of an infinite random data process. In Shapiro (2011), it is assumed that the random data process is stage-wise independent implying that the probability of a given random realization at a given stage does not depend on the random realization observed in the previous stage. Moreover, the distributions of the random events occurring between two consecutive stages are assumed to be known in advance. That is, not only are the types of the distributions known in advance, but the parameters are also fixed. Another assumption is that the optimal decision at a specific stage depends on only the random realization observed right before and the action taken at the preceding stage. That is, the history of random realizations does not have an influence on the decision maker's preference.

In our problem setting, the mean number of potential customers in the market in each period of the selling horizon is estimated in advance. As explained in Section 2, the mean demand observed by a given retailer in a given period depends on the retail price the manufacturer sets. This means that although we know that the distribution of the demand observed by each retailer is Poisson, the manufacturer's pricing decision influences its mean. Furthermore, since the inventory levels of the retailers are determinants of how the manufacturer will react, the retailers' starting inventory levels at the beginning of each period have to be reckoned with. Although the actions taken by the manufacturer in two consecutive periods are dependent because of the non-increasing price assumption, fortunately, the random data process is still stage-wise independent given a feasible set of actions taken by the manufacturer from the beginning of the selling horizon till the end. The differences in our model require the ideation of a new algorithm which is a variant of SDDP algorithm that is capable of handling the price-dependent infinite random event space. Through the variant SDDP algorithm, we will be able to analyze the case where the retail price set by the manufacturer in a given period has an influence on the demand distributions. Otherwise, the SDDP algorithm proposed and extended over time in the literature is not capable of dealing with this case. It will be explained below how the original SDDP algorithm proposed under some certain assumptions is adapted to the specifications of our decision-making process.

SDDP algorithm is proposed to solve sample average approximation problem (SAA) as explained in Shapiro (2011). The algorithm is executed iteratively and involves the consecutive implementations of two steps on each iteration. These steps are called backward step and forward step. The backward step is initialized by generating a number of realizations from the random event distributions in purpose for simulating the random behavior. Likewise, we have to generate a chosen number of Poisson demand realizations observed by the manufacturer-controlled retailer and the independent retailer in each period to build the SAA problem. Since the mean demand observed by a given retailer depends on the manufacturer's pricing decision, we cannot know the exact mean value in advance. For that reason, this fact obstructs the generation of ordered pairs of demand realizations for each period beforehand. However, we can simply generate realizations for the number of potential customers in the market for each period because the estimated mean aggregate demand is known in advance by problem definition. This implies that we can construct the SAA problem by substituting the expectation of the conditional expectation of the pre-decision profit-to-go function of the following period given the number of potential customers over all possible numbers of potential customers for the post-decision profit-to-go function of the current period in the objective function of the mathematical model of each period. For a given period t, this relation is as follows:

$$\varphi_t\left(PRIL_t^m, PRIL_t^r, p_t\right) = \mathbb{E}_{A_t}\left[\mathbb{E}_{D_t}\left[Q_{t+1}\left(PRIL_t^m, PRIL_t^r, p_t, \left(D_t^m, D_t^r\right)\right)\backslash A_t = a\right]\right]. \tag{4.1}$$

In the equation shown above, $PRIL_t^m$ and $PRIL_t^r$ stand for the manufacturer-controlled retailer's and the independent retailer's inventory levels right after the replenishments at the beginning of the period t. If M_t realizations are generated for the number of potential customers observed in the market in a given period t, then the approximate post-decision profit-to-go function ($\widetilde{\varphi}_t(PRIL_t^m, PRIL_t^r, p_t)$) of that period is expressed as follows:

$$\widetilde{\varphi}_t\left(PRIL_t^m, PRIL_t^r, p_t\right) = \frac{1}{M_t}\cdot\sum_{k=1}^{M_t}\mathbb{E}_{D_t}\left[\widetilde{Q}_{t+1}\left(PRIL_t^m, PRIL_t^r, p_t, \left(D_t^m, D_t^r\right)\right)\backslash A_t = a_{t,k}\right]. \tag{4.2}$$

In Equation (4.2) shown above, $a_{t,k}$ stands for the kth realization generated for the number of potential customers observed in period t. The approximate pre-decision profit-to-go function ($\widetilde{Q}_t(I_{t-1}^m, I_{t-1}^r, p_{t-1}, D_{t-1})$) of a given period t except the first period is characterized by the optimal value of the nonlinear SAA model shown below given the retailers' inventory levels (I_{t-1}^m and I_{t-1}^r) after replenishments at the beginning of period *t-1*, the retail price (p_{t-1}) of the product set by the manufacturer in period *t-1* and the ordered pair (D_{t-1}) of demands observed by the retailers in period *t-1*.

$$\begin{aligned}\widetilde{Q}_t\left(I_{t-1}^m, I_{t-1}^r, p_{t-1}, D_{t-1}\right) &= \max_{F_t} PRD_t\left(I_{t-1}^m, I_{t-1}^r, p_{t-1}, D_{t-1}^m, D_{t-1}^r\right)\\ &+ POD_t\left(I_{t-1}^r, p_t, D_{t-1}^r\right)\\ &+ \widetilde{\varphi}_t\left(IL_t^m\left(I_{t-1}^m, D_{t-1}^m\right), IL_t^r\left(I_{t-1}^r, D_{t-1}^r\right), p_t\right),\end{aligned} \tag{4.3}$$

where

$$F_t = \{p_t \in \mathbb{R} : p_t \le p_{t-1}, p_t \ge 0\}, \tag{4.4}$$

$$\widetilde{\varphi}_t\left(IL_t^m\left(I_{t-1}^m, D_{t-1}^m\right), IL_t^r\left(I_{t-1}^r, D_{t-1}^r\right), p_t\right) =$$

$$= \frac{1}{M_t} \cdot \sum_{k=1}^{M_t} \mathbb{E}_{D_t}\left[\widetilde{Q}_{t+1}\left(IL_t^m\left(I_{t-1}^m, D_{t-1}^m\right), IL_t^r\left(I_{t-1}^r, D_{t-1}^r\right), p_t, \left(D_t^m, D_t^r\right)\right) \backslash A_t = a_{t,k}\right], \tag{4.5}$$

$$IL_t^m\left(I_{t-1}^m, D_{t-1}^m\right) = max\left\{S_t^m, I_{t-1}^m - D_{t-1}^m\right\}, \tag{4.6}$$

$$IL_t^r\left(I_{t-1}^r, D_{t-1}^r\right) = max\left\{S_t^r, I_{t-1}^r - D_{t-1}^r\right\}. \tag{4.7}$$

In the nonlinear model presented above, $PRD_t(.)$ and $POD_t(.)$ are the functional representations of the pre-decision profit made right before the pricing decision for period t and the post-decision profit made right after the pricing decision in the current period t, respectively. The SAA model to be solved to determine the approximate optimal price to be set in the first period of the selling horizon is as follows:

$$\max_{F_1} FPP(p_1) + \widetilde{\varphi}_1\left(S_1^m, S_1^r, p_1\right), \tag{4.8}$$

where

$$F_1 = \{p_1 \in \mathbb{R} : p_1 \geq 0\}. \tag{4.9}$$

In the nonlinear model presented above, $FPP(.)$ stands for the functional representation of the profit made right after the replenishments at the beginning of the selling horizon. Despite the new formulation of the problem, it is still hard and most of the time impossible to derive a closed-form expression for the approximate post-decision profit-to-go functions. Therefore, we have to derive a function of the decision variable which returns an upper bound over the corresponding approximate post-decision profit-to-go function of each period. In this case, if we mean to derive an upper bound over the approximate post-decision profit-to-go function of a given period t given the retailers' inventory levels right after the replenishments at the beginning of the corresponding period, we have to derive an upper bound function for the conditional expectation of the approximate pre-decision profit-to-go function of the succeeding period $t + 1$ given each realization generated for the number of potential customers in period t at the beginning of the backward step and then take the sample average of these functions. This relation is as follows:

$$\widetilde{\varphi}_t\left(PRIL_t^m, PRIL_t^r, p_t\right) \leq \overline{\varphi}_t\left(PRIL_t^m, PRIL_t^r, p_t\right), \tag{4.10}$$

where

$$\overline{\varphi}_t\left(PRIL_t^m, PRIL_t^r, p_t\right) = \frac{1}{M_t} \cdot \sum_{k=1}^{M_t} \mathbb{E}_{D_t}\left[\widetilde{Q}_{t+1}\left(PRIL_t^m, PRIL_t^r, p_t, D_t\right) \backslash A_t = a_{t,k}\right]. \tag{4.11}$$

Since these two retailers are assumed to follow order-up-to inventory replenishment policy, $PRIL_t^m$ ranges between S_t^m and the maximum of all the order-up-to levels from the beginning of the selling horizon to period t. The same applies to $PRIL_t^r$ implying that there are finitely many pairs of inventory levels for which we have to find an upper bound for the approximate post-decision profit-to-go function of period t.

Given the number of potential customers in the market in an arbitrary period t, the amount of demand observed by either retailer in that period is trinomially distributed. The probability that a potential customer prefers purchasing the product from a given retailer is that retailer's price-dependent market share by problem definition. Let $MS_t^m(p_t)$, $MS_t^r(p_t)$ and $MS_t^o(p_t)$ stand for the manufacturer-controlled retailer's market share, the independent retailer's market share and the probability that a potential customer chooses no-purchase option given the price p_t set by the manufacturer in period t, respectively. Then, Equation (4.11) can be expanded as follows:

$$\overline{\varphi}_t\left(PRIL_t^m, PRIL_t^r, p_t\right) = \frac{1}{M_t} \cdot \sum_{k=1}^{M_t} \sum_{l=0}^{a_{t,k}} \sum_{m=0}^{a_{t,k}-l} \left(\overline{Q}_{t+1}\left(PRIL_t^m, PRIL_t^r, p_t, (l, m)\right)\right.$$

$$\cdot \binom{a_{t,k}}{l} \cdot \binom{a_{t,k} - l}{m} \cdot \left(MS_t^m(p_t)\right)^l \cdot \left(MS_t^r(p_t)\right)^m \cdot \left(MS_t^o(p_t)\right)^{a_{t,k}-l-m}, \tag{4.12}$$

where

$$\widetilde{Q}_{t+1}\left(PRIL_t^m, PRIL_t^r, p_t, (l, m)\right) \leq \overline{Q}_{t+1}\left(PRIL_t^m, PRIL_t^r, p_t, (l, m)\right). \tag{4.13}$$

As can be seen in the equation presented above, we have to derive an upper bound function for the pre-decision profit-to-go function of the succeeding period $t + 1$ given the inventory levels after replenishments in period t and for each possible pair of demands observed by the retailers in period t given each realization generated for the number potential customers in the market in period t. Afterwards, we have to take the sample average of all the upper bound functions multiplied by the trinomial mass function over all the realizations generated for the number of potential customers in period t at the beginning of the backward step. The quality of the upper bound dramatically impacts how fast the algorithm converges. Therefore, it necessitates solving Lagrangian Dual (LD) problem to derive a favorable upper bound function given an arbitrary price.

Since it is hard to solve the LD problem parametrically, an implementable policy consisting of the prices set throughout the selling horizon has to be selected. That is, if the length of the selling horizon is N periods, the implementable policy is defined as a set of N feasible prices. Let the trial decision in period t be denoted by $\overline{p_t}$. The criterion to be satisfied in the selection of an implementable policy is the feasibility implying that the decreasing price environment requires the relationship shown below:

$$\overline{p_N} \leq \overline{p_{N-1}} \leq \overline{p_{N-2}} \leq \cdots \leq \overline{p_1}. \tag{4.14}$$

We do not need to derive an upper bound function for the post-decision profit-to-go function of the last period by solving the LD problem because we can derive the closed-form expression. Such a derivation is possible because as explained before, we know the exact closed-form profit function of the dummy period $N + 1$. Then, the only upper bound imposed on each iteration over the post-decision profit-to-go function of the last period is:

$$\overline{\varphi}_N\left(PRIL_N^m, PRIL_N^r, p_N\right) = \frac{1}{M_N} \cdot \sum_{k=1}^{M_N} \sum_{l=0}^{a_{N,k}} \sum_{m=0}^{a_{N,k}-l} \left(Q_{N+1}\left(PRIL_N^m, PRIL_N^r, p_N, (l, m)\right)\right.$$

$$\cdot \binom{a_{N,k}}{l} \cdot \binom{a_{N,k} - l}{m} \cdot \left(MS_N^m(p_N)\right)^l \cdot \left(MS_N^r(p_N)\right)^m \cdot \left(MS_N^o(p_N)\right)^{a_{t,k}-l-m}. \tag{4.15}$$

Then, we will move on to the penultimate period and derive upper bound functions for the approximate post-decision profit-to-go function for the retailers' all possible inventory levels after replenishments at the beginning of the period. As explained before, given each feasible pair of inventory levels, we have to solve the LD problem of the last period to derive an upper bound function for the pre-decision profit-to-go function of the last period for each possible pair of demands observed by the retailers given a realization generated for the number of potential customers in the market in the penultimate period. Then, we have to repeat the same operation for all the other realizations generated for period *N-1*. When solving the LD problem, we assume that the price set by the manufacturer in period *N-1* is the trial decision $\overline{p}_{N-1}$. The LD problem to be solved to derive an upper bound for the pre-decision profit-to-go function given a feasible pair $\left(I_{N-1}^m, I_{N-1}^r\right)$ of inventory levels, a feasible pair $\left(D_{N-1}^m, D_{N-1}^r\right)$ of demands observed by the retailers and the trial decision $\overline{p}_{N-1}$ is as follows:

$$\min_{\lambda \geq 0} d(\lambda), \tag{4.16}$$

where

$$d(\lambda) = \max_{(p_N, T) \in F_N} POD_N\left(I_{N-1}^r, p_N, D_{N-1}^r\right) + T + \lambda \cdot \left(\overline{p}_{N-1} - p_N\right), \tag{4.17}$$

$$F_N = \left\{ (p_N, T) \in \mathbb{R}^2 : T \leq \overline{\varphi}_N \left(IL_N^m \left(I_{N-1}^m, D_{N-1}^m \right), IL_N^r \left(I_{N-1}^r, D_{N-1}^r \right), p_N \right), p_N \geq 0 \right\}. \tag{4.18}$$

There are some methods proposed to solve LD problem such as subgradient algorithm, Bundle's method, outer linearization etc. Let λ^* be the optimal solution of the LD problem formulated above. The next step is to find an upper bound for an arbitrary price p_{N-1} set by the manufacturer in the previous period. Since λ^* is always a feasible solution, the optimal solution of the Lagrangian relaxation problem shown below provides a part of the upper bound function for an arbitrary price p_{N-1}.

$$d(\lambda^*) = \max_{(p_N,T)\in F_N} POD_N \left(I_{N-1}^r, p_N, D_{N-1}^r \right) + T - \lambda^* . p_N, \tag{4.19}$$

where

$$F_N = \left\{ (p_N, T) \in \mathbb{R}^2 : T \leq \overline{\varphi}_N \left(IL_N^m \left(I_{N-1}^m, D_{N-1}^m \right), IL_N^r \left(I_{N-1}^r, D_{N-1}^r \right), p_N \right), p_N \geq 0 \right\}. \tag{4.20}$$

If we add the functional terms of the price set by the manufacturer in period *N-1* to the optimal solution of the Lagrangian relaxation problem shown above, then it provides an upper bound function of the price (p_{N-1}) set in the penultimate period over the approximate pre-decision profit-to-go function of the last period given the retailers' post-replenishment inventory levels (I_{N-1}^m and I_{N-1}^r) in period *N-1* and the amounts of demand (D_{N-1}^m and D_{N-1}^r) observed by the retailers in period *N-1*. That upper bound can be expressed as follows:

$$\overline{Q}_N \left(I_{N-1}^m, I_{N-1}^r, p_{N-1}, \left(D_{N-1}^m, D_{N-1}^r \right) \right) = PRD_N \left(I_{N-1}^m, I_{N-1}^r, p_{N-1}, D_{N-1}^m, D_{N-1}^r \right) + d(\lambda^*) + \lambda^* . p_{N-1}. \tag{4.21}$$

We have to repeat the same operations for all possible pairs of demands given each realization generated for the number of potential customers. Then, using Equation (4.12), we can derive the first upper bound function over the approximate post-decision profit-to-go function of period *N-1* for the pair $\left(I_{N-1}^m, I_{N-1}^r \right)$ of inventory levels. Likewise, we can derive upper bound functions for all the other feasible pairs of inventory levels.

In the process of finding an upper bound function for the approximate post-decision profit-to-go function of period *N-2* given the inventory levels after replenishments, as explained before, we have to derive upper bound functions for the approximate pre-decision profit-to-go function of period *N-1* given the inventory levels after replenishments in period *N-2* and all possible pairs of demands observed by the retailers in period *N-2* given each realization generated for the number of potential customers in period *N-2.* Given the inventory levels right after the replenishments and the pair of demands observed by the retailers in period *N-2,* we can calculate the inventory levels right after the replenishments in period *N-1.* Then, before solving the LD problem of period *N-1,* we have to impose the upper bound found for the approximate post-decision profit-to-go function of period *N-1* that corresponds to the inventory levels after the replenishments in period *N-1* by adding a constraint to the constraint set. This means that the constraint set of each problem will contain an extra constraint forcing the upper bound over the approximate post-decision profit-to-go function. Let $u_t^k \left(I_t^m, I_t^r, p_t \right)$ denote the upper bound function of p_t derived in the *kth* iteration for the approximate post-decision profit-to-go function of period *t* given the pair $\left(I_t^m, I_t^r \right)$ of inventory levels after the replenishments in period *t.* Then, at the end of the backward step of the first iteration, we have to solve the following problem to obtain the first provisional approximate optimal price for the first period and an upper bound over the optimal value of the SAA problem.

$$\max_{(p_1,T)\in F_1} FPP(p_1) + T, \tag{4.22}$$

where

$$F_1 = \left\{ (p_1, T) \in \mathbb{R}^2 : T \leq u_1^1 \left(S_1^m, S_1^r, p_1 \right) p_1 \geq 0 \right\}. \tag{4.23}$$

Let $\overline{\vartheta_1}$ be the upper bound and (p_1^1, T^*) be the optimal solution attained by solving the problem shown above. If the manufacturer fixes the price at p_1^1, then the expected total profit across all possible demand scenarios bounds the actual optimal value from below. We still do not know whether the optimal solution of the SAA problem we have been solving is p_1^1 or not. This means that pricing the product at p_1^1 in the first period, we can also acquire a lower bound for the optimal value of the SAA problem. In this case, we have to check how close the upper bound $\overline{\vartheta_1}$ and the lower bound are to one another. However, it might be hard to determine the lower bound since the larger number of realizations generated for the number of potential customers in each period at the beginning of the backward step, the larger number of demand scenarios for which to solve the models in a forward fashion. Therefore, we have to commit ourselves to constructing a one-sided confidence interval of the expected total profit made across all the demand scenarios comprising the realizations generated for the SAA problem. The forward step is intended for the construction of the confidence interval.

There are two alternative ways devised for the forward step. Assume that *M* realizations have been generated for the number of potential customers to simulate the random event of each period. In this case, there are a total of M^N scenarios. Since these realizations are sampled from the corresponding Poisson distributions, we know that the probability of observing either scenario among these M^N scenarios is $\frac{1}{M^N}$. Let $\mu_i(p_1^1)$ be the total profit given the *i*th scenario and the provisional optimal price p_1^1 attained at the end of the backward step. Then, the expected total profit across all scenarios of the SAA problem is computed as follows:

$$\mu(p_1^1) = \sum_{i=1}^{M^N} \frac{1}{M^N} . \mu_i(p_1^1). \tag{4.24}$$

As explained before, Equation (4.24) provides us with a lower bound for the optimal value of the SAA problem. In the first alternative way of finding a lower bound, we uniformly extract *K* scenarios from M^N scenarios that are composed of the realizations generated at the beginning of the algorithm. Then, we have to solve the models for each subscenario of these *K* demand scenarios using the trial solution p_1^1 of the first period and the corresponding upper bounds determined in the backward step. A subscenario of a given scenario consists of the feasible distributions of the potential customers to the retailers. Let $s_{[i]}^j$ be the number of ways of distributing the potential customers in period *j* given the *i*th selected scenario. Then, this scenario contains $\prod_{k=1}^N s_{[i]}^k$ subscenarios implying that we have to solve the models moving forward as many times for that specific scenario. Let $a_{t,[i]}$ denote the number of potential customers in period *t* given the *i*th selected scenario, $md_{t,[i]}^j$ be the demand observed by the manufacturer-controlled retailer in period *t* given the *j*th subscenario of the *i*th selected scenario, $rd_{t,[i]}^j$ be the demand observed by the independent retailer in period *t* given the *j*th subscenario of the *i*th selected scenario and $p_{t,[i]}^j$ be the optimal price in period *t* given the *j*th subscenario of the *i*th selected scenario and the provisional optimal price p_1^1 of the first period. Then, the probability of observing the subscenario *j* given that the *i*th scenario has materialized is given by:

$$P(S_i = j) = \prod_{t=1}^{N} \left(\begin{array}{c} a_{t,[i]} \\ md_{t,[i]}^j \end{array} \right) . \left(\begin{array}{c} a_{t,[i]} - md_{t,[i]}^j \\ rd_{t,[i]}^j \end{array} \right) . \left(MS_t^m \left(p_{t,[i]}^j \right) \right)^{md_{t,[i]}^j}$$

$$.\left(MS_t^r\left(p_{t,[i]}^j\right)\right)^{rd_{t,[i]}^j}.\left(MS_t^o\left(p_{t,[i]}^j\right)\right)^{a_{t,[i]}-md_{t,[i]}^j-rd_{t,[i]}^j}. \tag{4.25}$$

In the process of calculating the expected total profit given a scenario, the probability of observing a subscenario has to be calculated as shown above. That is, we have to keep track of the probabilities in a dynamic way since the price directly influences the probability that a potential customer purchases the product from each retailer. Considering the chosen K scenarios, we need a total of $\sum_{i=1}^{K}\left(\prod_{k=1}^{N}s_{[i]}^k\right)$ iterations in the forward step. Obviously, the sample average of the expected total profits made out of K scenarios given the price p_1^1 serves as an unbiased estimator of $\mu(p_1^1)$. Let $\mu_{[i]}(p_1^1)$ be the expected total profit made given the ith selected scenario. Then, the one-sided confidence interval with a confidence level of 1-α is as follows:

$$\sum_{i=1}^{K}\frac{1}{K}.\mu_{[i]}(p_1^1) - z_\alpha.\frac{\sqrt{\left(\sum_{j=1}^{K}\left(\mu_j(p_1^1)-\sum_{k=1}^{K}(1/K).\mu_{[k]}(p_1^1)\right)^2\right)\Big/K-1}}{\sqrt{K}} \leq \mu(p_1^1). \tag{4.26}$$

The left-hand side of the inequality 4.26 is a lower bound for the optimal value of the SAA problem. Let LB_1 denote the lower bound obtained at the end of the forward step of the first iteration. Then, if the inequality 4.27 shown below is satisfied given an acceptably small tolerance ϵ, then p_1^1 is the approximate optimal price to be set in the first period. Otherwise, we have to move on to the second iteration and derive new upper bounds for the post-decision profit-to-go functions using one of the feasible trial solutions obtained in the forward step.

$$\overline{\vartheta_1} - LB_1 < \epsilon. \tag{4.27}$$

Alternatively, let $\mu_i^j(p_1^1)$ be the profit made if the jth subscenario of the ith scenario occurs and $P_i^j(p_1^1)$ be the probability of observing the jth subscenario of the ith scenario given the price p_1^1 set by the manufacturer in the first period of the selling horizon. $P_i^j(p_1^1)$ is exactly the probability of observing the ith scenario multiplied by the conditional probability of observing the jth subscenario given the ith scenario. The conditional probability is computed calling the equation (4.25) and by definition of the SAA problem, the probability of observing a scenario is $\frac{1}{M^N}$. If the scenario i has s_i subscenarios, then the expected total profit across all realizations generated at the beginning of the backward step given the price p_1^1 is provided by:

$$\mu(p_1^1) = \sum_{i=1}^{M^N}\sum_{j=1}^{s_i}P_i^j(p_1^1).\mu_i^j(p_1^1). \tag{4.28}$$

As explained before, Equation (4.28) provides us with a lower bound for the optimal value of the SAA problem. In the second alternative way of finding a lower bound, we uniformly extract K demand subscenarios from a total of $\sum_{i=1}^{M^N}s_i$ subscenarios in the fashion that one of the M^N scenarios is uniformly chosen and then one of the subscenarios of that scenario is extracted. The extraction of the subscenario entails solving the models from the first period to the last one and dealing with the evolution of probabilities as in the previous alternative way. Firstly, we have to calculate the retailers' market shares in the first period given the provisional optimal price p_1^1 and then generate a binomial random variate standing for the demand observed by the manufacturer-controlled retailer in the first period for the corresponding scenario. Then, we have to subtract that random variate from the number of potential customers in the first period given the selected scenario. Then, we have to calculate the probability of a potential customer purchasing the product from the independent retailer given that it does not purchase from the manufacturer-controlled retailer. Then, we have to generate a binomial random variate standing for the demand observed by the independent retailer in the first period. Inserting the generated binomial random variates into the model of the second period, we have to solve it to find the optimal price to be set in that period given the scenario and its subscenario. Afterwards, we have to calculate the retailers' market shares in the second period given the optimal price that has just been attained. Then, we have to generate random variates as we do for the first period. At the end, this process forms a complete subscenario for the previously chosen scenario. Considering the chosen K scenarios, we need a total of K iterations in the forward step. Although the first alternative way possibly provides us with a better lower bound, the second way is superior in terms of running time because we are in control of the number of iterations needed in the forward step considering the exponentially growing number of iterations necessitated by the first way. Let $\mu_{[i]}(p_1^1)$ be the total profit made out of the ith selected subscenario. Then, $\sum_{i=1}^{K}\frac{1}{K}.\mu_{[i]}(p_1^1)$ is an unbiased estimator of (p_1^1). Then, the one-sided confidence interval with a confidence level of 1-α is as follows:

$$\sum_{i=1}^{K}\frac{1}{K}.\mu_{[i]}(p_1^1) - z_\alpha.\frac{\sqrt{\left(\sum_{j=1}^{K}\left(\mu_j(p_1^1)-\sum_{k=1}^{K}(1/K).\mu_{[k]}(p_1^1)\right)^2\right)\Big/K-1}}{\sqrt{K}} \leq \mu(p_1^1). \tag{4.29}$$

The left-hand side of the inequality 4.29 is a lower bound for the optimal value of the SAA problem. Let LB_1 denote the lower bound obtained at the end of the forward step of the first iteration. Then, if the inequality 4.30 shown below is satisfied given an acceptably small tolerance ϵ, then p_1^1 is the approximate optimal price to be set in the first period. Otherwise, we have to move on to the second iteration and derive new upper bounds for the post-decision profit-to-go functions using one of the feasible trial solutions obtained in the forward step.

$$\overline{\vartheta_1} - LB_1 < \epsilon. \tag{4.30}$$

As explained before, if we are currently carrying out the kth iteration, we have to derive an extra upper bound for the approximate post-decision profit-to-go function of each period for all possible inventory levels after replenishment. That is, we have to keep all the upper bounds derived in the previous iterations. After implementing the backward step and the forward step, we obtain a new upper bound and a new lower bound for the optimal value of the SAA problem, respectively. If the stopping criterion is satisfied at the end of the iteration k, then p_1^k is the approximate optimal solution of the SAA problem for the first period of the selling horizon. The step-by-step summary of the variant SDDP algorithm is provided in Appendix A.

In the following section, four critically important contractual parameters are evaluated to observe how the changes in each parameter impact the approximate optimal price, the retailers' market shares and their expected total net profits. Some approaches on the selection of the best compromise values of these contractual parameters are also provided. For all these analyses, the variant SDDP algorithm proposed in this section is implemented and the manufacturer is assumed to specify its pricing strategy based on the approximate optimal solutions returned by this algorithm.

5. Numerical experiment

We have to implement the algorithm designed in the previous section for some problem instances to observe the manufacturer's pricing decisions through this approach. There exist some critically pivotal contractual parameters such as reimbursement rate, discount rate, markdown rate and refund per product returned by the independent retailer. Therefore, it is fundamental to analyze the evolution of the manufacturer's pricing strategy over the varying values of these parameters. It is out of scope to determine the optimal values for these contractual parameters but it can be beneficial to get an insight into the selection process for some further research. At this point, the selected values for these contractual parameters have to satisfy both retailers' profit expectations by enabling them to recover their initial outlay on the production or the acquisition of some initial inventory at the beginning of the selling horizon. However, before solving any problem instance, we have to choose a suitable method to estimate the retailers' market shares given the price set by the manufacturer in a given period.

One of the most widely used methods employed to estimate the market shares for existing alternatives in a given market is multinomial logit models (MNL) first proposed by Luce (1959) with the derivation of choice probabilities. Then, Luce and Suppes (1965) show the connection between the logit choice probability functions and the unobserved utility distributed extreme value. Finally, McFadden (1974) finishes off the research by proving that logit choice probability functions always entail the extreme value distribution of the unobserved utility. In this section, we utilize customized forms of the choice probability functions that are built on a price-dependent utility function. As explained before, there are three alternatives a potential customer can select among in our problem setting. Therefore, there exist three choice probability functions associated with a purchase from the manufacturer-controlled retailer, a purchase from the independent retailer and no-purchase option. The probability of a potential customer purchasing the product from the manufacturer-controlled retailer is given by.

$$MS_m(p) = \frac{exp^{\frac{\mu_m - p}{\tau}}}{exp^{\frac{\mu_m - p}{\tau}} + exp^{\frac{\mu_r - (1-\gamma).p}{\tau}} + 1}. \tag{5.1}$$

Likewise, the probability of a potential customer purchasing the product from the independent retailer is.

$$MS_r(p) = \frac{exp^{\frac{\mu_r - (1-\gamma).p}{\tau}}}{exp^{\frac{\mu_m - p}{\tau}} + exp^{\frac{\mu_r - (1-\gamma).p}{\tau}} + 1}. \tag{5.2}$$

If a potential customer does not prefer purchasing this product from either retailer, then it directly leaves the market since there is no substitute good. The proportion of the number of potential customers choosing the no-purchase option to the number of potential customers in the market is provided by.

$$MS_o(p) = \frac{1}{exp^{\frac{\mu_m - p}{\tau}} + exp^{\frac{\mu_r - (1-\gamma).p}{\tau}} + 1}. \tag{5.3}$$

As can easily be observed, all the probabilities presented above add up to *1*. In these choice probability functions, μ_m and μ_r stand for the mean maximum price a potential customer is willing to pay to purchase the product from the manufacturer-controlled retailer and the independent retailer, respectively. The maximum-willingness-to-pay values can be elicited from a sample of potential customers by administering a marketing survey to them. In such a survey, some information on delivery times, post-sale services etc. should be provided about the retailers and each potential customer should be asked about the maximum price it is willing to pay to purchase the product from each one of the retailers. Then, the mean maximum price can be estimated for each retailer by the sample average of the maximum-willingness-to-pay values provided by the potential customers.

Another parameter showing up in the choice probability functions is the scale factor τ. In Luce and Suppes (1965) and McFadden (1974), it is shown that the unobserved utility follows extreme value distribution and the difference between two extreme value random variables is logistically distributed. The variance of a logistic random variable is $\frac{\pi^2.\tau^2}{6}$ so we have to estimate this variance to obtain an estimator of the scale factor. For this purpose, we have to compute the sample variance of the maximum-willingness-to-pay values provided by the potential customers for both the retailers. Then, we can extract the estimator of the scale factor by equating the sample variance with the variance of the logistic random variable.

In this section, we observe the influence of the changes in the value of each critically significant contractual parameter on the approximately optimal price that the manufacturer should set in the first period given the pricing strategy we propose by solving the mathematical models shown in Section 3 for a selling horizon of three periods. We also provide some approaches for the selection process of the values of these contractual parameters to render the price protection contract profitable and favorable for both of the retailers. We have done an extreme value analysis for the other parameters than the contractual parameters discussed throughout this section. We have observed that the changes in the value of those parameters have no influence on how the approximately optimal price and the retailers' true expected total profits change in relation to the discussed contractual parameters. The changes in the value of those parameters impact only the amount of change in the approximately optimal price and the retailers' expected total profits. For that reason, we fix those parameters at specific values and do not change them throughout the section. The values that those parameters take on are presented in Table 5.1 shown below.

Apart from contractual and non-contractual parameters, there exist some parameters the values of which we have to fix to implement the variant SDDP algorithm. We generate 15 Poisson random variates standing for the number of potential customers in the market for each period of the selling horizon. We form 100 demand subscenarios in the forward step as explained in the previous section to obtain a lower bound for the optimal value of the actual SAA problem. We also have to define the stopping criterion to check whether to stop implementing the algorithm at the end of each iteration or not. As the stopping criterion, we would like the upper bound obtained at the end of the backward step to be in 10 % neighborhood of the absolute value of the lower bound obtained at the end of the backward step.

The first contractual parameter we examine is the reimbursement rate. As defined before, reimbursement rate is the proportion of the independent retailer's on-hand inventory that is eligible for reimbursement in case of a reduction in the price set by the manufacturer in a given period. Although it seems to take on values ranging between 0 % and 100 %, it might also take on a value strictly larger than 100 % under some special circumstances. Therefore, we solve problem instances for various values of the reimbursement rate by setting discount rate to 40 %, markdown rate to 20 % and refund per returned product to 200$. The influence of the changes in the reimbursement rate on the approximately optimal price the manufacturer sets, the retailers' market shares and the proportion of the lost customers in the first period is shown in Table 5.2 for reimbursement rates not larger than 100 % and in Table 5.3 for reimbursement rates above 100 %.

We can easily deduce from Table 5.2 and Table 5.3 that as the reimbursement rate increases, the manufacturer tends to reduce the price. The first reason for such a tendency is that the manufacturer avoids possibly higher amounts of reimbursement in the following periods of the selling horizon. The manufacturer raises the independent retailer's market share by diminishing the price so as to boost demand the independent retailer observes because higher amount of demand

Table 5.1
The values of fixed contractual and non-contractual parameters.

Parameter	Value
Holding cost per dollar per period ($)	0.05
Discount rate for backordered demand (%)	15
Salvage value ($)	60
Production costs ($)	(60, 60, 60)
Manufacturer-controlled retailer's reorder points	(22, 19, 17)
Independent retailer's reorder points	(15, 13, 10)
Mean number of potential customers per period	(22, 19, 15)
Mean maximum-willingness-to-pay values for retailers ($)	(200, 175, 140)
Multinomial logit scale factors	(32.66, 27.45, 25.55)
Allowable amounts of backordered demand for retailers	(15, 15, 0)

Table 5.2
The influence of the reimbursement rate up to 100% on the approximate optimal price and market shares.

Reimbursement rate (%)	40	60	70	80	100
Approximate optimal price ($)	176.37	176.22	176.14	176.06	175.89
Proportion of lost customers (%)	10.95	10.91	10.89	10.87	10.83
Manufacturer's market share (%)	22.57	22.60	22.61	22.63	22.65
Independent retailer's market share (%)	66.48	66.49	66.50	66.50	66.52

Table 5.3
The influence of the reimbursement rate above 100% on the approximate optimal price and market shares.

Reimbursement rate (%)	150	400	550	700	850	1300
Approximate optimal price ($)	175.44	172.14	169.36	166.38	163.75	154.93
Proportion of lost customers (%)	10.71	9.92	9.29	8.66	8.13	6.56
Manufacturer's market share (%)	22.73	23.28	23.74	24.23	24.66	26.08
Independent retailer's market share (%)	66.56	66.80	66.97	67.11	67.21	67.36

means lower amount of on-hand inventory held by the independent retailer at the end of the period. Another reason is that as the price decreases, the manufacturer-controlled retailer's market share also increases. In this case, the demand observed by the manufacturer-controlled retailer is on the upswing, which attenuates the negative impact stemming from the decreasing selling price.

Since the product in question is a slow-moving *A* item, the reorder points are relatively low. In this case, reimbursement rates below 100 % might be unsatisfactory for the independent retailer because of a possibly low amount of remaining on-hand inventory it holds at the end of each period. Therefore, the manufacturer and the independent retailer might negotiate reimbursement rates above 100 % at the beginning of the selling horizon to make the price protection contract more appealing for the independent retailer. However, both parties have to be pleased with the expected total profit they make throughout the selling horizon. For this reason, we also have to observe the effect of the changes in the reimbursement rate on the expected total profits. For this purpose, we form 100 demand subscenarios by generating demand realizations from the true distributions in the forward step just as we do to obtain a lower bound for the optimal value of the SAA problem. However, the way how the demand realizations are generated is different in this case.

Given the price set by the manufacturer in the first period, we can easily calculate the retailers' market shares employing the aforementioned equations (5.1) and (5.2). Since the number of potential customers in a given period is Poisson distributed, the demand observed by each retailer is also Poisson distributed. Then, we can determine the mean demand observed by a given retailer multiplying the known mean number of potential customers in the first period by its market share derived from the corresponding choice probability function. This means that when sequentially solving the mathematical models, we have to generate a Poisson random variate representing the demand observed by each retailer given the price set in the given period. In this way, we draw a sample of 100 profit values for each retailer. Then, we construct one-sided and two-sided confidence intervals of the expected total profit made by each retailer with a confidence level of 80 % to observe the impact of various reimbursement rates.

As can be observed in Figure 5.1, as the reimbursement rate rises, the bounds over the independent retailers' expected total net profit increase until the reimbursement rate reaches a specific value and then they start decreasing. This means that extremely high reimbursement rates are less favorable than moderate reimbursement rates. If it is more highly weighted how much profit the independent retailer makes throughout the selling horizon, then the best choice turns out to be 400 % among all the tried alternatives. The reimbursement rates of 150 % and 550 % return almost the same results and they are not dominated by the reimbursement rate of 400 %. This means that they are also good choices for the independent retailer's profitability.

The evolution of the bounds over the manufacturer's expected total net profit is shown in Figure 5.1. The confidence intervals of the manufacturer's expected total profit are mostly overlapping until the reimbursement rate reaches 70 %. We do not observe any dramatic shift of the manufacturer's expected total profit as the value taken on by the reimbursement rate ranges between 100 % and 850 %. We can state that the changes in the value of reimbursement rate do not have an enormous influence on the manufacturer's expected total profit. Therefore, it is less critical to reckon with the manufacturer's minimum allowable expected total profit than the independent retailer's minimum allowable expected total profit when selecting a compromise value for the reimbursement rate. If it is more highly weighted how much profit the manufacturer-controlled retailer makes throughout the selling horizon, then the best choice turns out to be 60 % among all the tried alternatives. The reimbursement rates of 40 %, 70 % and 150 % are also some good options for the manufacturer's profitability. If both parties lean towards relinquishing some revenue, then the reimbursement rates of 150 % and 70 % can be good options as a compromise solution. Of course, one of the most important things is to settle on the minimum allowable expected profit for each retailer because it also has an influence on the best compromise solution.

Another significant contractual parameter to be assessed is markdown rate. We solve problem instances for various feasible values of markdown rate by setting discount rate to 60 %, reimbursement rate to 70 % and refund per returned product to 200$. The influence of the changes in the markdown rate on the approximately optimal price the manufacturer sets, the retailers' market shares and the proportion of the lost customers in the first period is shown in Table 5.4.

We can infer from Table 5.4 shown above that as the markdown rate rises, the manufacturer proposes to increase the price. Since higher markdown rates mean higher market shares for the independent retailer and the manufacturer controlled retailer's market share is negatively correlated, the manufacturer's goal is to counteract the financial loss arising from the decreasing market share by increasing the price. At the same time, since the independent retailer's market share rises, the amount of the remaining on-hand inventory the independent retailer holds at the end of the period is expected to be relatively low. Therefore, the manufacturer also tends to reduce the reimbursement cost. In order to observe the impact of markdown rate on the retailers' expected total net profits, we draw a sample of 100 profit values for each retailer in the same way as we do for the reimbursement rate. Then, we construct one-sided and two-sided confidence intervals of the expected total profit made by each retailer with a confidence level of 80 %.

As can be seen in Figure 5.2, as markdown rate increases, the confidence interval of the manufacturer's expected total net profit shifts down until the markdown rate reaches a specific value and then it stars shifting up. The reason for such a trend is that the increase in the price and the amounts of replenishment orders placed by the independent retailer counterbalance the negative impact of the decrease in the manufacturer's market share at a specific value of the markdown rate.

As markdown rate increases, the confidence interval of the independent retailer's expected total net profit shifts down as shown in Figure 5.2 because the price at which the independent retailer sells the product to the end customer decreases although its market share

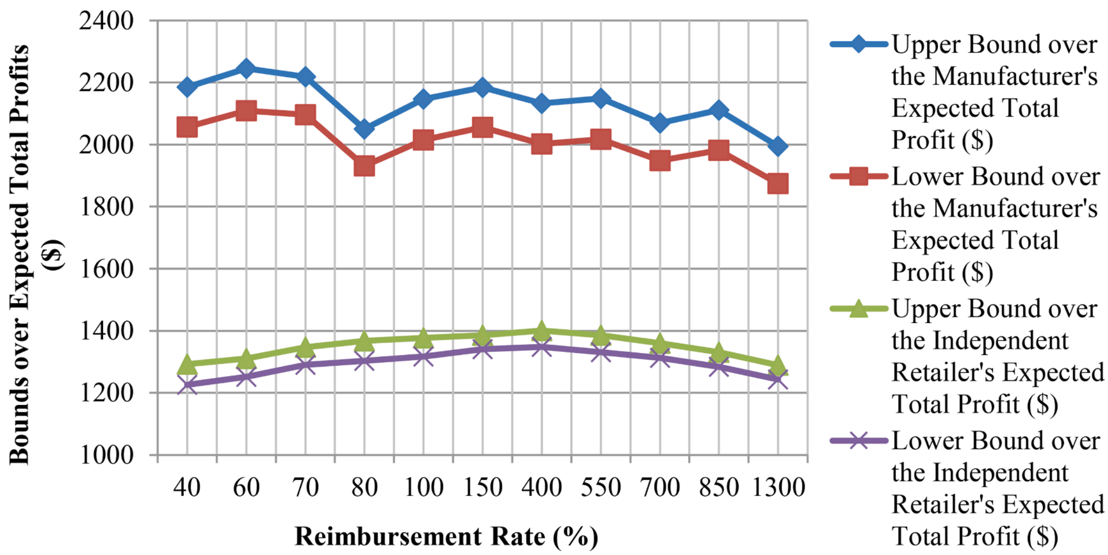

Fig. 5.1. The evolution of the bounds of expected total net profits in relation to reimbursement rate.

Table 5.4
The influence of markdown rate on the approximate optimal price and market shares.

Markdown rate (%)	0	10	20	30	40	50
Approximate optimal price ($)	158.31	160.49	163.53	169.39	183.52	216.84
Proportion of lost customers (%)	12.24	10.17	8.08	6.39	5.45	5.32
Manufacturer's market share (%)	43.88	34.10	24.69	16.31	9.03	3.50
Independent retailer's market share (%)	43.88	55.73	67.23	77.30	85.52	91.18

increases. It seems to be the best option that both retailers sell the product at the same price by setting the markdown rate to 0 % since it provides the highest profit for the manufacturer and the possibly highest profit for the independent retailer. The markdown rate of 10 % is almost as profitable for the independent retailer as the markdown rate of 0 %. However, the increase of the markdown rate from 0 % to 10 % drastically cuts down the manufacturer's expected total net profit. We can also deduce that the independent retailer makes larger profit than the manufacturer does until the markdown rate reaches 50 %. Therefore, if the profit made by the manufacturer is more highly weighted, there are no many options to choose from.

If the discount rate is fixed at 40 %, then there are more options in which the manufacturer makes larger profit than the independent retailer does. In case the discount rate is set to 40 %, the bounds of the one-sided and two-sided confidence intervals of the manufacturer's and the independent retailer's expected total net profits are as shown in Table 5.5. As can be seen in the table, the markdown rates of 0 % and 10 % dominate the markdown rates of 20 % and 30 % since the increase in the markdown rate from 10 % weighs down both retailers' profitability. If a profit of around 1610$ is sufficient for the independent retailer, then the markdown rate of 10 % can be chosen since at that value, the manufacturer seems to make the possibly highest profit. Otherwise, a markdown rate between 0 % and 10 % has to be selected for the independent retailer to make higher profit but in that case, the manufacturer has to renounce some of its revenue. This means that the minimum allowable expected total net profits and the profit made by which retailer is more highly weighted are very critical evaluation measures in

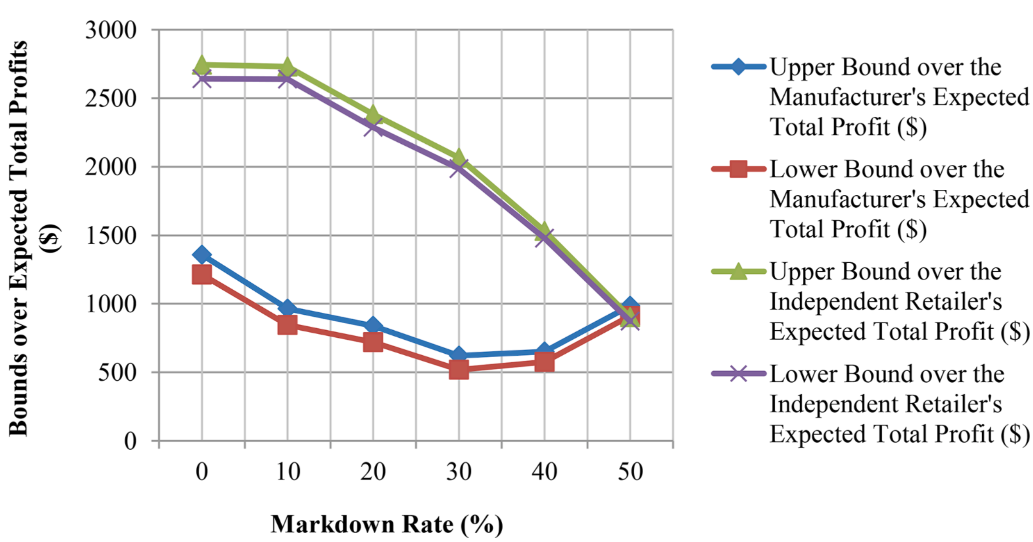

Fig. 5.2. The evolution of the bounds of the expected total net profits in relation to markdown rate.

Table 5.5
The influence of the markdown rate on the bounds of the confidence intervals of the manufacturer's and the independent retailer's expected total net profits in case of the discount rate of 40%.

Markdown rate (%)	0	10	20	30
Upper bound of two-sided CI of manufacturer's expected total profit ($)	2133.8	2300.8	2218.9	2217.1
Lower bound of two-sided CI of manufacturer's expected total profit ($)	1978.5	2154	2095.9	2090.3
Lower bound of one-sided CI of manufacturer's expected total profit ($)	2005.2	2300.8	2117	2112.4
Upper bound of two-sided CI of independent retailer's expected total profit ($)	1948.4	1674.3	1346.7	780.4
Lower bound of two-sided CI of independent retailer's expected total profit ($)	1880.9	1610.9	1290.5	733.1
Lower bound of one-sided CI of independent retailer's expected total profit ($)	1892.5	1621.8	1300.2	741.2

the selection process of the markdown rate to determine the best compromise solution.

The third contractual parameter of which the retailers have to compromise on the value is discount rate. Just as in the analyses of the preceding two contractual parameters, we solve problem instances for various feasible values of discount rate by setting markdown rate to 10 %, reimbursement rate to 70 % and refund per returned product to 200$. The influence of the changes in the discount rate on the approximately optimal price the manufacturer sets, the retailers' market shares and the proportion of the lost customers in the first period is shown in Table 5.6.

As we can observe from Table 5.6, the manufacturer lessens the price as the discount rate goes up. The manufacturer increases its market share by taking this action, thereby observing higher demand. Likewise, the independent retailer's market share also increases as the discount rate increases implying that the replenishment orders placed by the independent retailer are larger in this case. Larger replenishment orders mean the independent retailer's less remaining on-hand inventory at the end of each period so the manufacturer reduces the reimbursement cost it incurs, as well. Therefore, the increase in the demand observed by the manufacturer and the increase in the amounts of replenishment orders placed by the independent retailer partly mitigates the negative impact of the decrease in the selling price and the discounted price offered to the independent retailer.

As discount rate increases, the confidence interval of the manufacturer's expected total net profit shifts down as can be observed in Figure 5.3. On the contrary, the confidence interval of the independent retailer's expected total net profit shifts up because although the marked down price at which it sells the product to the end customer decreases, the downturn in the discounted price offered by the manufacturer for

Table 5.6
The influence of discount rate on the approximate optimal price and market shares.

Discount rate (%)	20	30	40	50	60	70
Approximate optimal price ($)	186.03	176.59	170.12	165.30	160.49	155.17
Proportion of lost customers (%)	19.07	15.23	12.98	11.51	10.17	8.86
Manufacturer's market share (%)	29.24	31.20	32.43	33.28	34.10	34.94
Independent retailer's market share (%)	51.69	53.57	54.59	55.21	55.73	56.20

replenishment orders is more precipitous. Therefore, the independent retailer's profit per product increases in this case. Furthermore, the rise in the discount rate also triggers a rise in the independent retailer's market share, as well and the opposite impact of the decreasing reimbursement revenue does not hurt the independent retailer too much.

The manufacturer's expected total net profit is negatively correlated with the independent retailer's expected total net profit as can be inferred from Figure 5.3. This means that there exists no dominated solution and there is a trade-off between the options. As the independent retailer's expected total net profit is higher than the manufacturer's expected total net profit at the discount rate of 50 %, the manufacturer's expected total net profit surpasses when the discount rate is set to 40 %. This means that there exists a balance point between these values. If the manufacturer's expected total net profit is more highly weighted, then the discount rate has to be below this balance point. On the contrary, if the independent retailer's expected total net profit is more highly weighted, then the discount rate has to be above the balance point. However, in the selection of the best compromise solution, the minimum allowable profits are also significant evaluation measures as explained before.

The last critical contractual parameter to be analyzed is the refund per product returned by the independent retailer at the end of the selling horizon. Just as in the analyses of the previous three contractual parameters, we solve problem instances for some values of refund by setting markdown rate to 80 %, reimbursement rate to 70 % and discount rate to 60 %. The influence of the changes in the refund on the approximately optimal price the manufacturer sets, the retailers' market shares and the proportion of the lost customers in the first period is shown in Table 5.7.

As we can observe from Table 5.7, the manufacturer increases the price as the refund per product rises until it reaches a specific value. That value seems to be between 50 and 100 in this case. Then, the approximate optimal price shows a decreasing pattern until the refund reaches another breaking point. That breaking point is above the first breaking point and less than 100 in this case. After the refund surpasses the second breaking point, the approximate optimal price seems to increase very slightly as the refund rises. After the second breaking point, the manufacturer tries to increase the revenue generated by selling the product to the end customer at higher prices in purpose for covering the increase in the refund per product. However, since the independent retailer's market share decreases as the price set by the manufacturer increases, the independent retailer is likely to be possessed of higher amount of remaining on-hand inventory at the end of the selling horizon. This might lead to a rise in total refund. For that reason, the manufacturer is conservative in pricing and avoids dramatic increases after the second breaking point.

After the second breaking point, the market shares fluctuate and the deviations are unnoticeably small. Since the low values of refund do not weigh on the manufacturer's profitability too much, the manufacturer tends to increase the price more steeply compared to the higher values of refund although it decreases the manufacturer's and the independent retailer's market shares.

As refund increases, the confidence interval of the manufacturer's expected total net profit shifts down as can be observed in Figure 5.4 since the total refund increases and the manufacturer's actions can partly compensate for that increase. Obviously, the most preferable value of refund is 0$ among the evaluated alternatives for the manufacturer but the independent retailer is possibly dissatisfied with this value since its expected total net profit is low in that case. On the contrary, the confidence interval of the independent retailer's expected total net profit shifts up. Apparently, the best choice is 250$ among the evaluated alternatives for the independent retailer but the manufacturer-controlled retailer might be dissatisfied this time since it is required to forgo some profit in that case. We can easily deduce that the balance point is not reached yet and refund can be increased more until the independent retailer's expected total net profit becomes level

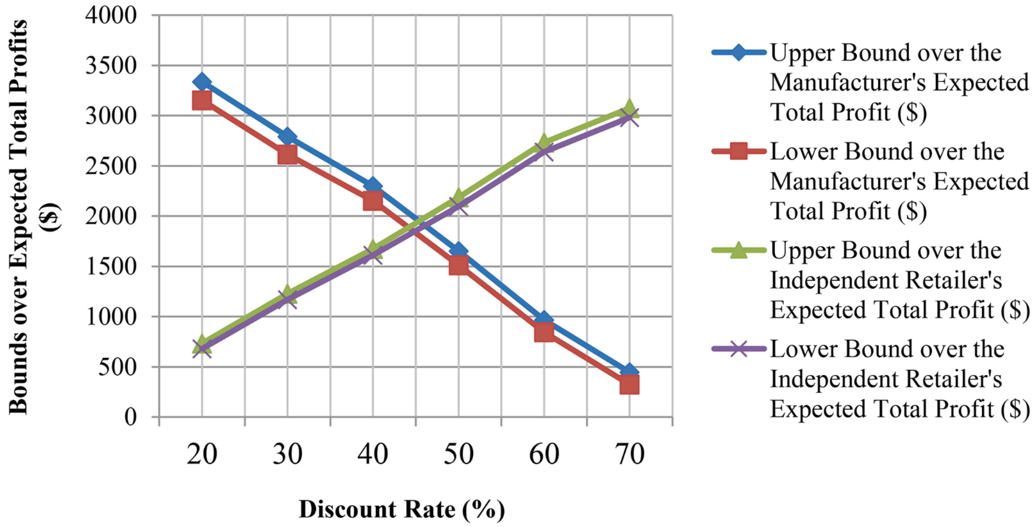

Fig. 5.3. The evolution of the bounds of the expected total net profits in relation to discount rate.

Table 5.7
The influence of refund on the approximate optimal price and market shares.

Refund ($)	0	50	100	150	200	250
Approximate optimal price ($)	185.0037	185.4432	176.1405	176.1416	176.1427	176.1437
Proportion of lost customers (%)	13.3392	13.4718	10.8913	10.8916	10.8919	10.8922
Manufacturer's market share (%)	21.1127	21.0375	22.6126	22.6124	22.6123	22.6121
Independent retailer's market share (%)	65.5481	65.4907	66.4961	66.4960	66.4958	66.4957

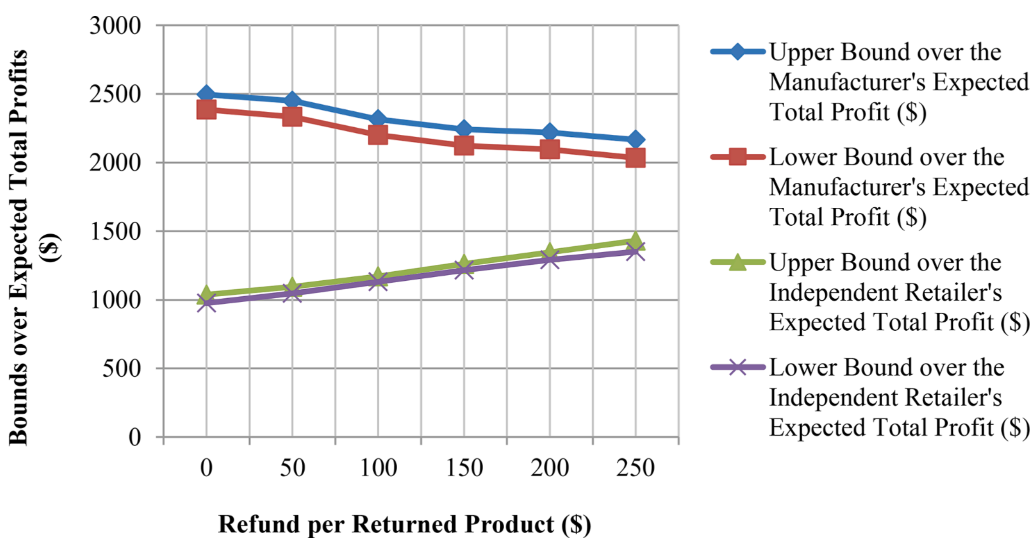

Fig. 5.4. The evolution of the bounds of the expected total net profits in relation to refund per returned product.

with the manufacturer's expected total net profit. If the manufacturer's profit is more highly weighted, then the refund has to be less than that balance point. Otherwise, the refund has to be more than or equal to the balance point. Of course, the minimum allowable profits are critical evaluation measures in search for the best compromise solution.

In conclusion, the algorithm takes an acceptably small number of iterations to converge. The maximum observed number of iterations in the data collection process is six and it has occurred only once. The running time of a single iteration of the algorithm has been around one and half an hour and we have observed reasonable amount of deviations when different combinations of values have been assigned to the problem parameters. The running time dwindles from iteration to iteration because of the extra upper bound constraints added to each constraint set in the backward step of the algorithm. Compared to the approximate dynamic programming algorithms proposed to get around three curses of dimensionality, the running time per iteration is longer since the algorithm traverses all the possible pairs of inventory levels with which the retailers can start a given period instead of visiting a single state on each iteration. However, it deals with the estimation of the post-decision profit-to-go functions of a given period in a more skillful way by deriving upper bound functions of retail price set in the previous period instead of visiting a single retail price on each iteration. Considering that approximate dynamic programming algorithms are bound to necessitate an undue number of iterations for a decent approximation, a longer running time per iteration of the variant SDDP algorithm is tolerable.

By the inferences from the analyses done in this section, the selection of a compromise value for the discount rate and the markdown rate is essential to ensure high profitability for both the manufacturer and the independent retailer given the approximately optimal pricing strategy proposed in this study. The changes in the values of reimbursement rate and refund per returned product do not have a massive impact on the manufacturer's expected total profit since the manufacturer can keep a tight grip on the market shares by updating its pricing strategy. However, the selection of ideal values for these two contractual parameters is significant to provide the independent retailer with high enough profitability so that it is convinced to keep the inventory of the product. The most critical thing is to avoid inordinately high values of reimbursement rate since the independent retailer sustains financial loss in that case. The retailers have to determine their minimum allowable profit values that they will stipulate in the contract negotiations because those values are very critical in the selection of the best compromise values of the contractual parameters. In the selection process, trade-offs have to be reckoned with scrupulously, as well. Furthermore, the weights assigned to the retailers' expected total net profits have a conspicuous impact on the best compromise values. Therefore, whether the price protection contract is as profitable for both parties as expected or not depends on the retailers' profit expectations implying that the suitable selection of the contractual parameter values and the accuracy in the requirements of the price protection contract are very decisive.

6. Summary and conclusion

In high-tech industry, products suffer rapid obsolescence because of perpetual technological developments and the frequent introduction of more advanced products. Therefore, they depreciate over time and sellers observe high demand uncertainty. Manufacturers prefer collaborating with some retailers to sell soon-to-be-obsolete products to a wide range of customers from lower-income segments. However, the depreciation of these products obliges sellers to reduce the price over time so retailers have to be convinced to stock this kind of products with some incentives. For this purpose, manufacturers offer retailers price commitment policies to protect them financially.

The effectiveness of price commitment policies in channel coordination and in the attainment of a win–win outcome for involved actors is widely discussed in the literature. However, generally, researchers commit themselves to studying cases where wholesale prices and retail prices are assumed to be fixed and known in advance implying that demand distributions are independent from pricing decisions with the purpose of determining optimal order policies. However, it is also intriguing whether or not the evaluated price commitment policies and return policies are capable of coordinating a supply chain and providing a win–win outcome in an environment where retail price is a decision variable and it has an influence on the demand distribution. As a starting point for such analyses, we commit ourselves to examining the impact of price protection and end-of-life return opportunity on the optimal retail price and the actors' profits. However, we do not deal with the determination of the optimal inventory replenishment policy and the optimal return policy.

In this paper, we focus on determining a manufacturer's optimal pricing strategy in a non-increasing price environment where a manufacturer-controlled retailer and an independent retailer sell a slow-moving *A* item. The distribution of the number of potential customers in the market is assumed to be Poisson since it is the best fit to the demand observed for slow-moving *A* items. These two retailers are committed to complying with the terms of a price protection contract by which they compromise on the values of some contractual parameters. The retailers are assumed to follow *(R, S)* inventory replenishment policy and the values of the inventory replenishment policy parameters are known in advance.

The retail price at which the independent retailer sells the product to the end customer is determined through Retail Fixed Markdown Policy. The wholesale price is dependent on the retail price set by the manufacturer. That price is calculated by discounting the retail price set by the manufacturer by a fixed rate. Furthermore, the manufacturer is committed to reimbursing the independent retailer for only a part of its on-hand inventory if it decreases the retail price in any period. The manufacturer also offers the independent retailer a refund for the returned products at the end of the selling horizon.

We model the problem through stochastic programming approach. If the mathematical model presented in Section 3 can be solved to optimality, then it provides the actual optimal pricing strategy. However, the solution process is troubled by three curses of dimensionality as explained in Section 4. The complexity of the model precludes the derivation of closed-form expressions for the post-decision profit-to-go functions. For that reason, an applicable approach is essential to specify an approximately optimal pricing strategy. For this purpose, we propose the variant SDDP algorithm to the manufacturer to determine its approximately optimal pricing strategy and analyze the impact of the changes in the values of critical contractual parameters on the approximately optimal retail price and the true expected total profits of both the manufacturer and the independent retailer given that pricing strategy.

We adapt the SDDP algorithm the implementation of which is extended and analyzed by Shapiro (2011) for the case of infinite random data process to our model in Section 4. Unlike in the SDDP algorithm proposed by Shapiro (2011), we take the conditional expectation of each pre-decision profit-to-go function given the number of potential customers in the market. In that way, we could generate demand realizations for the number of potential customers since the mean number of potential customers is assumed to have been estimated and to be known in advance. In this case, we derive upper bound functions for the conditional expectations of the pre-decision profit-to-go functions and take the sample average of those functions over the generated realizations to derive the upper bound functions for the post-decision profit-to-go functions. The original SAA problem solved in Shapiro (2011) is linear but the SAA problem we solve is nonlinear. For that reason, we employ Lagrangian duality instead of LP duality when deriving the upper bound functions. Furthermore, we draw a sample of demand scenarios by solving the models sequentially in the forward step unlike the way it is done in Shapiro (2011) because the mean demand observed by each retailer in a given period depends on the price set by the manufacturer.

We implement the algorithm for some values of reimbursement rate, discount rate, markdown rate and refund per returned product. The algorithm takes at most six iterations to converge. The amount of time that each iteration takes depends on the number of realizations generated at the beginning of the backward step. However, the size of the sample drawn in the forward step does not have a substantial impact on the running time. The findings on the impact of an increase in the value of each critical contractual parameter on approximately optimal price and the retailers' true expected total net profits are summarized in Table 6.1 shown below.

The inferences that we have drawn are as follows:

- The selection of compromise values for all the critical contractual parameters necessitates setting the manufacturer's and the independent retailer's minimum allowable expected total profit values,
- The selection of a compromise value for the discount rate and the markdown rate is essential to ensure high profitability for both the manufacturer and the independent retailer given the approximately optimal pricing strategy proposed in this study,
- The changes in the values of reimbursement rate and refund per returned product do not have a massive impact on the manufacturer's expected total profit since the manufacturer can keep a tight grip on the market shares by updating its pricing strategy,
- The selection of compromise values for reimbursement rate and refund per returned product is significant to provide the independent retailer with high enough profitability,

Table 6.1
Impact of an increase in the values of the critical contractual parameters on approximately optimal price and the retailers' true expected total net profits.

	Approximately optimal price	CI of manufacturer's expected total net profit	CI of independent retailer's expected total net profit
Reimbursement rate	Slightly decreasing	Alternating	First shifting up, then shifting down
Markdown rate	Increasing	Following trajectory or inverted trajectory	Shifting down
Discount rate	Decreasing	Shifting down	Shifting up
Refund	First alternating, then slightly increasing	Shifting down	Shifting up

- The critical thing is to avoid inordinately high values of reimbursement rate since the independent retailer sustains financial loss in that case,
- Compared to the approximate dynamic programming algorithms proposed to get around three curses of dimensionality, the running time per iteration is longer since the algorithm traverses all the possible pairs of inventory levels with which the retailers can start a given period instead of visiting a single state on each iteration,
- Considering that approximate dynamic programming algorithms are bound to necessitate an undue number of iterations for a decent approximation because of the clumsy way how the value functions are updated from iteration to iteration, a longer running time per iteration of the variant SDDP algorithm is tolerable.

The retailers' expected total net profits can also be weighted differently and in that way, they can look for the best compromise solution. This approach also refers to multi-objective optimization. The objective functions of the models presented in Section 3 can be reformed and the algorithm proposed in Section 4 can be implemented to determine approximately Pareto optimal pricing strategy. The assumptions can be relaxed to study some different cases. For example, we can relax the assumption that the retailers follow *(R,S)* inventory replenishment policy and allow them to determine their own replenishment policies. In this case, we have to optimize the amount of replenishment orders, as well. In this way, we can determine the optimal inventory replenishment policies and check whether or not the supply chain can be coordinated in presence of a price commitment policy that comprises price protection and end-of-life return opportunity in a selling environment where retail and wholesale prices are not fixed. We can also allow the independent retailer to set the retail price at which it sells the product to the end customer by excluding RFM policy. In both cases, we have to determine Nash-equilibrium values so a different solution methodology has to be proposed. We can also analyze a case where the manufacturer collaborates with more than two independent retailers and a case where a fast-moving *A* item or a *B* item is sold throughout the selling horizon. We can also study a problem in which a manufacturer collaborates with some retailers to sell multiple products. The effectiveness of some other privileges such as mid-life return opportunities, special discount policies etc. in channel coordination and in the achievement of a win–win outcome can also be analyzed. The existence of a lead time can also impact retailers' optimal inventory replenishment policies so we can also study a case where there exists a fixed or random lead time before the delivery of replenishment orders. In a much more advanced problem setting, a methodology can be proposed for continuous pricing and replenishment instead of periodic pricing and replenishment. Furthermore, the variant SDDP algorithm can be improved in different aspects to reduce the number of iterations needed till convergence and the running time of each iteration. Some other existing methodologies can be evaluated in terms of whether they are applicable and adaptable to the problem studied in this paper and after the adaptation of another methodology, we can compare the variant SDDP algorithm to that new method in terms of efficiency.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

Appendix A

Step 0. Initialization:

Step 0a. Initialize the number of demand realizations M_j to be generated for all.

$j \in \{1,2,\cdots,N\}$

Step 0b. Generate a Poisson random variate $a_{j,m}$ with a mean of μ_j for all.

$m \in \{1,2,\cdots,M_j\}, j \in \{1,2,\cdots,N\}$

Step 0c. Initialize the trial decision $\overline{p_j}$ for all $j \in \{1,2,\cdots,N\}$ satisfying:

$0 \le \overline{p_N} \le \overline{p_{N-1}} \le \cdots \le \overline{p_1}$

Step 0d. Set $LF^0_{j,l,n} = \left\{ \left(p_j, T\right) \in \mathbb{R}^2 : p_j \ge 0 \right\}$ for all $j \in \{1,2,\cdots,N\}$,

$l \in \left\{1,2,\cdots,\max_{j\in\{1,2,\cdots,N\}} S^m_j\right\}, n \in \left\{1,2,\cdots,\max_{j\in\{1,2,\cdots,N\}} S^r_j\right\}$

Step 0e. Set $\overline{\vartheta}_N(l,n,p) = \frac{1}{M_N}.\sum_{x=1}^{M_N}\sum_{y=0}^{a_{N,x}}\sum_{z=0}^{a_{N,x}-y} Q_{N+1}(l,n,p,(y,z))$.

$.\left(\frac{a_{N,x}}{y}\right).\left(\frac{a_{N,x}-y}{z}\right).\left(MS^m_N(p)\right)^y.\left(MS^r_N(p)\right)^z.\left(MS^o_N(p)\right)^{a_{N,x}-y-z}.$

Step 0f. Set k = 1.

Step 1. Set $\overline{\vartheta}_t(l,n,p) = 0$ for all $t \in \{1,2,\cdots,N-1\}$.

Step 2. Do for $s = S^m_{N-1}, S^m_{N-1}+1, \dots, \max_{j\in\{1,2,\cdots,N-1\}} S^m_j$;

$t = S^r_{N-1}, S^r_{N-1}+1, \dots, \max_{j\in\{1,2,\cdots,N-1\}} S^r_j$:

Step 2a. Do for $u = 1,2,\dots,M_{N-1}$; $v = 0,1,\dots,a_{N-1,u}$; $z = 0,1,\dots,a_{N-1,u}$ -v:

Step 2aa. Set $mi = \max\{s\text{-}v, S^m_N\}$ and $ri = \max\{t\text{-}z, S^r_N\}$.

Step 2ab. Set $LF^k_{N,mi,ri} = LF^{k-1}_{N,mi,ri} \cap \{(p_N, T) \in \mathbb{R}^2 : T \le \overline{\vartheta}_N(mi, ri, p_N)\}$.

Step 2ac. Solve the Lagrangian dual problem:

$$\min_{\lambda\geqslant 0} \max_{(p_N,T)\in LF^k_{N,mi,ri}} POD_N(t, p_N, z) + T + \lambda.(\overline{p}_{N-1} - p_N)$$

and let λ^* be the optimal solution of the Lagrangian dual problem.

Step 2ad. Solve:

$$d^* = \max_{(p_N,T)\in LF^k_{N,mi,ri}} POD_N(t, p_N, z) + T - \lambda^*.p_N$$

Step 2ae. Update $\overline{\vartheta}_{N-1}(s,t,p)$ using:

$$\overline{\vartheta}_{N-1}(s,t,p) = \overline{\vartheta}_{N-1}(s,t,p) + \frac{1}{M_{N-1}}.\binom{a_{N-1,u}}{v}.\binom{a_{N-1,u}-v}{z}.$$

$$.\left(MS^m_{N-1}(p)\right)^v.\left(MS^r_{N-1}(p)\right)^z.$$

$$.\left(MS^o_{N-1}(p)\right)^{a_{N-1,u}-v-z}.$$

$$.\left(PRD_N(s,t,p,v,z) + d^* + \lambda^*.p\right)$$

Step 2b. Set $u^k_{N-1}(s,t,p) = \overline{\vartheta}_{N-1}(s,t,p)$.

Step 3. Do for $j = N\text{-}2, N\text{-}3, \dots, 1$; $s = S^m_j, S^m_j + 1, \dots, \max_{g\in\{1,2,\cdots,j\}} S^m_g$;

$t = S^r_j, S^r_j + 1, \dots, \max_{g\in\{1,2,\cdots,j\}} S^r_g$:

Step 3a. Do for $u = 1, 2,\dots,M_j$; $v = 0,1,\dots,a_{j,u}$; $z = 0,1,\dots,a_{j,u}$ -v:

Step 3aa. Set $mi = \max\{s\text{-}v, S^m_{j+1}\}$ and $ri = \max\{t\text{-}z, S^r_{j+1}\}$.

Step 3ab. Set.$LF^k_{j+1,mi,ri} = LF^{k-1}_{j+1,mi,ri} \cap$

$$\cap\left\{(p_{j+1}, T) \in \mathbb{R}^2 : T \le u^k_{j+1}(mi, ri, p_{j+1})\right\}$$

Step 3ac. Solve the Lagrangian dual problem:

$$\min_{\lambda\geqslant 0} \max_{(p_{j+1},T)\in LF^k_{j+1,mi,ri}} POD_{j+1}(t, p_{j+1}, z) + T + \lambda.(\overline{p}_j - p_{j+1})$$

and let λ^* be the optimal solution of the Lagrangian dual problem.

Step 3ad. Solve:

$$d^* = \max_{(p_{j+1},T)\in LF^k_{j+1,mi,ri}} POD_{j+1}(t, p_{j+1}, z) + T - \lambda^*.p_{j+1}.$$

Step 3ae. Update $\overline{\vartheta}_j(s,t,p)$ using:

$$\overline{\vartheta}_j(s,t,p) = \overline{\vartheta}_j(s,t,p) + \frac{1}{M_j}.\binom{a_{j,u}}{v}.\binom{a_{j,u}-v}{z}.\left(MS^m_j(p)\right)^v.$$

$$.\left(MS^r_j(p)\right)^z.\left(MS^o_j(p)\right)^{a_{j,u}-v-z}.$$

$$.\left(PRD_{j+1}(s,t,p,v,z) + d^* + \lambda^*.p\right)$$

Step 3b. Set $u^k_j(s,t,p) = \overline{\vartheta}_j(s,t,p)$.

Step 4. Determine a new candidate approximately optimal retail price for the first period:

Step 4a. Set $LF^k_{1,S^m_1,S^r_1} = LF^{k-1}_{1,S^m_1,S^r_1} \cap \{(p_1, T) \in \mathbb{R}^2 : T \le u^k_1(S^m_1, S^r_1, p_1)\}$.

Step 4b. Solve:

$$\max_{(p_1,T)\in LF^k_{1,S^m_1,S^r_1}} FPP(p_1) + T$$

and let (p^k_1, T^*) be the optimal solution and $\overline{\vartheta_k}$ be the optimal value of the.
problem.

Step 5. Uniformly choose K demand scenarios from the entire set of $\prod_{i=1}^N M_i$ scenarios with replacement and let D^w_j be the number of potential customers in the market in period j given wth demand scenario.

Step 6. Set $\delta_w = FPP(p^k_1)$ for all $w \in \{1, 2, \cdots, K\}$.

Step 7. Do for $w = 1,2,\dots,K$:

Step 7a. Set $mi_1 = S_1^m$ and $ri_1 = S_1^r$.

Step 7b. Do for $t = 1,2,\ldots,N$:

Step 7ba. Calculate the manufacturer-controlled retailer's and the independent retailer's market shares in the first period given the price p_t^k using the market share functions and let $MS_t^m(p_t^k)$ and $MS_t^r(p_t^k)$ be the manufacturer-controlled retailer's market share and the independent retailer's market share, respectively.

Step 7bb. Generate a binomial random variate od_m representing the demand observed by the manufacturer-controlled retailer given the probability of success of $MS_t^m(p_t^k)$ and the sample size of D_t^w.

Step 7bc. Generate a binomial random variate od_r representing the amount of demand observed by the independent retailer given the probability of success of $\frac{MS_t^r(p_t^k)}{1-MS_t^m(p_t^k)}$ and the sample size of $D_t^w - od_m$.

Step 7bd. If $t \neq N$, update the retailers' inventory levels after replenishments in the following period $t + 1$:

$mi_{t+1} = \max\{mi_t\text{-}od_m, S_{t+1}^m\}$ and $ri_{t+1} = \max\{ ri_t\text{-}od_r, S_{t+1}^r\}$.

Step 7be. If $t \neq N$, solve:

$$\max_{(p_{t+1},T)\in F_{t+1}} POD_{t+1}(ri_t, p_{t+1}, od_r) +$$

$$+PRD_{t+1}\left(mi_t, ri_t, p_t^k, od_m, od_r\right) + T$$

where

$$F_{t+1} = LF^k_{t+1,mi_{t+1},ri_{t+1}} \cap \left\{(p_{t+1}, T) \in \mathbb{R}^2 : p_{t+1} \leq p_t^k\right\}$$

and let $\left(p_{t+1}^k, T^*\right)$ be the optimal solution.

Step 7bf. If $t \neq N$, update the total profit by using:

$$\delta_w = \delta_w + POD_{t+1}\left(ri_t, p_{t+1}^k, od_r\right) +$$

$$+PRD_{t+1}\left(mi_t, ri_t, p_t^k, od_m, od_r\right).$$

Step 7bg. If $t = N$, update the total profit by using:

$$\delta_w = \delta_w + Q_{N+1}\left(mi_N, ri_N, p_N^k, (od_m, od_r)\right)$$

Step 8. Construct a one-sided confidence interval with a confidence level of $1-\alpha$ to determine a lower bound LB_k over the optimal value of the actual SAA problem:

$$LB_k = \sum_{w=1}^{K} \frac{1}{K}.\delta_w - z_\alpha.\frac{\sqrt{\left(\sum_{j=1}^{K}\left(\delta_j - \sum_{w=1}^{K}(1/K).\delta_w\right)^2\right)\Big/K-1}}{\sqrt{K}}$$

Step 9. If $\overline{\vartheta_k} - LB_k < \epsilon$, then terminate the algorithm. p_1^k is the approximately optimal retail price for the first period.

Step 10. If $\overline{\vartheta_k} - LB_k \geq \epsilon$, then:

Step 10a. Set $\overline{p_j} = p_j^k$ for all $j \in \{1, 2, \cdots, N\}$.

Step 10b. Set $k = k + 1$ and return to Step 1.

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