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Solution of jamming transition problem using adomian decomposition method

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Abstract

Purpose – The purpose of the study is to obtain an analytical approximate solution for jamming transition problem (JTP) using Adomian decomposition method (ADM).

Design/methodology/approach – In this study, the jamming transition is presented as a result of spontaneous deviations of headway and velocity that is caused by the acceleration/breaking rate to be higher than the critical value. Dissipative dynamics of traffic flow can be represented within the framework of the Lorenz scheme based on the car-following model in the one-lane highway. Through this paper, an analytical approximation for the solution is calculated via ADM that leads to a solution for headway deviation as a function of time.

Findings – A highly nonlinear differential equation having no exact solution due to JTP is considered and headway deviation is obtained implementing a number of different initial conditions. The results are discussed and compared with the available data in the literature and numerical solutions obtained from a built-in numerical function of the mathematical software used in the study. The advantage of using ADM for the problem is presented in the study and discussed on the basis of the results produced by the applied method.

Originality/value – This is the first study to apply ADM to JTP.

Keywords Adomian decomposition method, Headway deviation, Jamming transition problem, Lorenz system, Traffic flow

Paper type Research paper

1. Introduction

In most of natural and unnatural networks (Nagatani, 1998, 2002), properties of the transported entities such as traffic flow are the primary interest of practical applications. Within this scope, the traffic congestion related to transportation networks is a practical interest and there have been many studies related to traffic congestion in various disciplines. (Carlier *et al.*, 2008; Sánchez-Medina *et al.*, 2010; Bauza and Gozálvez, 2013; Lu *et al.*, 2013; Celikoglu, 2013; Zhang *et al.*, 2014; Qu *et al.*, 2015; Khan *et al.*, 2017). There is a largest admissible capacity of a node in a highway network, such that traffic congestion occurs when the volume of traffic flow exceeds this value. Such nodes may be a bottleneck in the

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network. The problem of traffic congestion is a multidisciplinary area studied by the experts of various disciplines including mathematics, engineering, urban planning and logistics.

Traffic flow can be modeled using different approaches classified as microscopic, macroscopic or kinetic (Hidas, 2005; Delitala and Tosin, 2007; Geroliminis and Sun, 2011; Li *et al.*, 2013; Garavello and Piccoli, 2017; Puppo *et al.*, 2017). Microscopic models deal with vehicle movements and interactions between vehicles such as free driving, car following and lane changing. However, macroscopic traffic models provide a measure of vehicle interactions in terms of collective variables. First-order models (Lighthill and Whitham, 1955; Richards, 1956) are one of the two common macroscopic models, describing traffic as a continuous function of the vehicle density and traffic velocity in space and time. The other model is the second-order model that includes an additional partial differential equation for traffic flow to adapt to changing conditions (Cremer, 1979; Payne, 1979). In kinetic models, a probability distribution function $f(t, x, V)$ is defined to express the position x of a vehicle at time t with velocity V.

Traffic jam is a microscopic phenomenon for which some researchers have focused on the jamming transition problem (JTP) in its various forms (Nagel, 1994; Nagatani, 1998, 2000, 2002; Ben-Naim and Krapivsky, 1999; Maerivoet and De Moor, 2005; Peng *et al.*, 2012; Gupta and Redhu, 2013, 2014; Xiao *et al.*, 2017). The problem has been considered using thermodynamic (Nagatani, 1998; Ge *et al.*, 2015), hydrodynamic and kinetic theories (Nagel, 1994; Peng *et al.*, 2011), based on the car-following model (Tang *et al.*, 2009; Gupta and Redhu, 2013; Yang *et al.*, 2013; Qian *et al.*, 2017a; Li *et al.*, 2017), Maxell model (Ben-Naim and Krapivsky, 1999) and cellular automaton model (Maerivoet and De Moor, 2005; Qian *et al.*, 2017b; Xiao *et al.*, 2017). There are also some works available transforming JTP into a nonlinear oscillator with a restoring damping term, via the Lorenz system (Ganji *et al.*, 2011, 2012; Han *et al.*, 2013).

There are also a number of numerical techniques for the problem of traffic congestion in traffic flow, often having restrictions for practical cases. Based on the difficulty in dealing with nonlinear problems, the use of a simple and straightforward solution technique with the minimum computing time is a great advantage to provide a solution to the problem. Various exact (He and Abdou, 2007; Ganji *et al.*, 2009), numerical (Anderson *et al.*, 1984) and semi-analytical (Seyed Alizadeh *et al.*, 2008; Barari, *et al.*, 2008; Ganji *et al.*, 2009; Hashemi Kachapi *et al.*, 2009; Joneidi *et al.*, 2009; Kimiaeifar *et al.*, 2009; Babaelahi *et al.*, 2010; Momeni *et al.*, 2010) techniques for solving nonlinear oscillatory systems are available in the literature.

In this study, an analytical approximation technique, namely, the Adomian decomposition method (ADM) (Adomian, 1980, 1994), is used to solve JTP. ADM has been successfully used for solving a large class of problems for the past three decades. ADM produces accurate solutions effectively and easily for linear and nonlinear, ordinary and partial and deterministic or stochastic differential equations with analytical continuous approximations converging accurate solutions rapidly. With the implementation of the method, linearization, perturbation or other restrictive techniques do not supply any advantage compared to ADM. This is a great advantage of the method because the assumptions used in the solution techniques may sometimes seriously change the behavior of the solution. ADM is well suited for physical problems and is selected for the solution of JTP in this paper, which is for the first time that the method is used for the solution of the problem. JTP was previously solved using a number of other techniques (Ganji *et al.*, 2011, 2012). Compared to methods producing solutions given at discrete points, ADM provides an analytical approximation, which is a continuous function with respect to the independent parameter for the governing equation. This solution is continuous, differentiable and integrable in the solution domain. In the following sections, the governing equation of the problem will be described at first and then ADM will be introduced and applied to the problem. After tabulating ADM solutions and comparing them with available solutions in the literature, the results obtained will be discussed in the conclusion part of the manuscript. In addition, after computing the solution, its advantages over the methods used previously will be evaluated in the text and practical implications of the numerical results will also be discussed.

2. Governing equations of jamming transition problem

Lorenz system is proposed for modeling JTP as a nonlinear non-conservative oscillator in terms of a third-order nonlinear differential equation. In this model, the jamming transition is presented as a result of spontaneous deviations of headway and velocity that is caused by the acceleration/breaking rate to be higher than the critical value. Dissipative dynamics of traffic flow can be represented within the framework of the Lorenz scheme based on the car-following model in the one-lane highway (Olemskoi and Khomenko, 2001, Khomenko *et al.*, 2004).

Based on this model, the system of equations given below is obtained:

$$\dot{\eta} = -\eta/t_\eta + v \tag{1}$$

$$\dot{v} = -v/t_v + g_v \tau \eta \tag{2}$$

$$\dot{\tau} = (\tau_0 - \tau)/t_\tau - g_\tau \eta v + \lambda(t) \tag{3}$$

In this equation, η is headway deviation, τ is velocity deviation, v is acceleration/braking time, t_η, t_v and t_τ, appropriate relaxation time, g_v and g_τ are positive constants and dot means differentiation with respect to time. The system of equations does not take the fluctuation of acceleration/braking time into account.

First terms on the right-hand side of equations between equations 1 and 3 represent the relaxation of each quantity to an equilibrium value. The second term on the right-hand side of equation (2) provides a positive feedback of headway deviation η and acceleration/breaking time τ on velocity deviation ν. Due to the positive feedback, velocity deviation will increase and this will be the reason for traffic jam formation. From equation (3), it may be observed that relaxation for acceleration/breaking time τ occurs to a finite value τ_0 rather than zero, which represents the time necessary for an automobile to reach a characteristic velocity.

Due to LeChatelier principle, to impede a jam formation, the headway deviation η and its velocity deviation ν should vary to prevent the growth of acceleration/breaking time τ, since the decrease of acceleration/breaking time assists to the formation of stable traffic flow (Khomenko *et al.*, 2004).

There is no analytical solution available for the system of equations. To obtain an acceptable solution, some simple assumptions are introduced such that $t_\eta \gg t_\tau$ and $t_\eta \approx t_v$, i.e. relaxation time for acceleration/breaking is very small when compared to relaxation times for headway deviation and its velocity. Due to these conditions, coordination of acceleration/breaking time τ occurs by the variation of headway and velocity deviations. The assumptions lead to $\dot{\tau} t_\tau \approx 0$ and equation (3) becomes,

$$\tau = \tau_0 - g_\tau t_\tau \eta v \tag{4}$$

Introducing t_η, $\eta_m = (g_v g_\tau t_\tau t_\eta)^{-1/2}$, $v_m = t_\eta^{-3/2}(g_v g_\tau t_\tau)^{-1/2}$, $\tau_c = (g_v t_\eta^{2})^{-1}$ as natural scale factors for time, headway deviation, velocity deviation and acceleration/braking time, respectively, and $g_v t_\tau t_\eta^2$ for noise term of the characteristic acceleration/braking time τ_0. Velocity deviation and its time derivative can be obtained from equation (1). Inserting these parameters and equation (4) into equation (2) provides the following scaled nonlinear stochastic oscillation equation (Khomenko *et al.*, 2004),

$$\ddot{\eta} + \dot{\eta}\left(1 + \sigma + \eta^2\right) - \eta(\varepsilon - \sigma) + \eta^3 = 0 \tag{5}$$

where $\sigma \equiv t_\eta/t_v$ and $\varepsilon \equiv \tau_0/\tau_c$. It is not possible to produce an analytical solution for equation (5) in which highly nonlinear terms exist. In such problems, approximate solution techniques must be used to provide a solution. Such methods may be numerical, variational or analytical approximate methods. In this study, an analytical approximate solution technique, i.e. ADM, is used to obtain an analytical approximation for JTP.

3. Adomian decomposition method

In the ADM, a differential equation of the following form is considered:

$$Lu + Ru + Nu = g(x) \tag{6}$$

where L is the linear operator which is highest-order derivative, R is the remainder of linear operator including derivatives of less order than L, Nu represents the nonlinear terms and g is the source term. Equation (6) can be rearranged as:

$$Lu = g(x) - Ru - Nu \tag{7}$$

The inverse operator L^{-1} is applied to both sides of equation (7) to obtain:

$$u = L^{-1}(g(x)) - L^{-1}(Ru) - L^{-1}(Nu) \tag{8}$$

After integrating source term $g(x)$ and combining it with the terms arising from boundary conditions of the problem, a function $f(x)$ is defined in equation (8) as follows:

$$u = f(x) - L^{-1}(Ru) - L^{-1}(Nu) \tag{9}$$

The nonlinear operator $Nu = F(u)$ is represented by an infinite series of specially generated (Adomian) polynomials for the specific nonlinearity. Assuming Nu is analytic, we write:

$$F(u) = \sum_{k=0}^{\infty} A_k \tag{10}$$

The polynomials A_k's are generated for all kinds of nonlinearity so that they depend only on u_0 to u_k components. These polynomials can be produced by the following algorithm:

$$A_0 = F(u_0) \tag{11}$$

$$A_1 = u_1 F'(u_0) \tag{12}$$

$$A_2 = u_2 F'(u_0) + \frac{1}{2!} u_1^2 F''(u_0) \tag{13}$$

$$A_n = \sum_{v=1}^{n} c(v,n) F^{(v)}(u_0) \tag{14}$$

where $c(v, n)$ are products of v components of u's whose subscripts sum to n-divided by the factorial of the number of repeated subscripts. Detailed information about the method can be found in the literature (Adomian, 1980, 1994). The solution of the problem, $u(x)$, is defined by the following series:

$$u = \sum_{k=0}^{\infty} u_k \tag{15}$$

where the components of the series are determined recursively as follows:

$$u_0 = f(x) \tag{16}$$

$$u_{k+1} = -L^{-1}(Ru_k) - L^{-1}(A_k), \qquad k \geq 0 \tag{17}$$

4. Application of Adomian decomposition method to jamming transition problem

ADM is applied to the problem by selecting the second derivative term for the linear operator in the formulation. Then, equation (18) is obtained:

$$\ddot{\eta} = -\left[(1+\sigma)\dot{\eta} + (\sigma - \varepsilon)\eta + \dot{\eta}\eta^2 + \eta^3\right] \tag{18}$$

Operator $L_{tt} = d^2/dt^2$ leads to its inverse operator as, $L_{tt}^{-1} = \int_0^t \int_0^t dtdt$. The inverse operator is then applied to both sides of equation (18):

$$L_{tt}^{-1} L_{tt}\eta = -L_{tt}^{-1}\left[(1+\sigma)\dot{\eta} + (\sigma - \varepsilon)\eta + \dot{\eta}\eta^2 + \eta^3\right] \tag{19}$$

Initial conditions for the problem are $\eta(0) = A$ and $\dot{\eta}(0) = 0$. These conditions are now introduced to formulation as below:

$$\eta - \eta(0) - t\dot{\eta}(0) = -L_{tt}^{-1}\left[(1+\sigma)\dot{\eta} + (\sigma - \varepsilon)\eta + \dot{\eta}\eta^2 + \eta^3\right] \tag{20}$$

$$\eta = A - L_{tt}^{-1}\left[(1+\sigma)\dot{\eta} + (\sigma - \varepsilon)\eta + \dot{\eta}\eta^2 + \eta^3\right] \tag{21}$$

Equation (21) is now decomposed according to its linear and nonlinear terms. To treat the nonlinear terms, Adomian polynomials should be used:

$$\eta = A - L_{tt}^{-1}\left[(1+\sigma)\dot{\eta} + (\sigma - \varepsilon)\eta\right] - L_{tt}^{-1}\left[\dot{\eta}\,\eta^2\right] - L_{tt}^{-1}\left[\eta^3\right] \tag{22}$$

Hence, the solution is arranged as follows by introducing Adomian polynomials for the nonlinear terms in equation (22):

$$\sum_{n=0}^{\infty} \eta_n(t) = \eta_0(t) + L_{tt}^{-1}\left(\sum_{n=0}^{\infty} A_n\right) + L_{tt}^{-1}\left(\sum_{n=0}^{\infty} B_n\right) \tag{23}$$

Initial approximation for the solution procedure is selected from equation (22) which produces a recurrence relation based on initial approximation as below:

$$\eta_0(t) = A, \tag{24}$$

$$\eta_{n+1}(t) = -L_{tt}^{-1}\left[(1+\sigma)\dot{\eta}_n + (\sigma - \varepsilon)\eta_n\right] - L_{tt}^{-1}\left(\sum_{n=0}^{\infty} A_n\right) - L_{tt}^{-1}\left(\sum_{n=0}^{\infty} B_n\right) \tag{25}$$

Adomian polynomials A_n and B_n in equation (25) for the first five approximations are as follows:

$$A_0 = \eta_0^2 \eta'_0 \tag{26}$$

$$A_1 = \eta_0\left(2\eta_1\eta'_0 + \eta_0\eta'_1\right) \tag{27}$$

$$A_2 = \left(\eta'_0\eta_1^2 + 2\eta_0\eta'_1\eta_1 + \eta_0\left(2\eta_2\eta'_0 + \eta_0\eta'_2\right)\right) \tag{28}$$

$$A_3 = \left(\eta'_1\eta_1^2 + 2\left(\eta_2\eta'_0 + \eta_0\eta'_2\right)\eta_1 + \eta_0\left(2\eta_3\eta'_0 + 2\eta_2\eta'_1 + \eta_0\eta'_3\right)\right) \tag{29}$$

$$A_4 = \left(\eta'_2\eta_1^2 + 2\left(\eta_3\eta'_0 + \eta_0\eta'_3\right)\eta_1 + \eta_2^2\eta'_0 + 2\eta_2\left(\eta_1\eta'_1 + \eta_0\eta'_2\right) + \eta_0\left(2\eta_4\eta'_0 + 2\eta_3\eta'_1 + \eta_0\eta'_4\right)\right) \tag{30}$$

$$B_0 = \eta_0^3 \tag{31}$$

$$B_1 = 3\eta_0^2\eta_1 \tag{32}$$

$$B_2 = 3\eta_0\left(\eta_1^2 + \eta_0\eta_2\right) \tag{33}$$

$$B_3 = \left(\eta_1^3 + 6\eta_0\eta_2\eta_1 + 3\eta_0^2\eta_3\right) \tag{34}$$

$$B_4 = 3\left(\eta_2\eta_1^2 + 2\eta_0\eta_3\eta_1 + \eta_0\left(\eta_2^2 + \eta_0\eta_4\right)\right) \tag{35}$$

The algorithm to produce Adomian polynomials has been given in equations 11-14. However, the reader may refer (Babolian and Javadi, 2004, Zhu *et al.*, 2005, Pourdavish, 2006) to get additional information on various techniques for calculating these polynomials. After conducting successive approximations with the expression given in equation (25), the headway deviation η can be obtained through ADM as an analytical expression, which is a continuous function in the solution domain.

5. Results and discussion

As case studies of the problem, previously available solutions are going to be used for comparison purposes. For this purpose, some numerical values are preselected from previous studies (Ganji *et al.*, 2011, 2012). Each set of different values of parameters is called a mode. Table I includes values for four modes to be used in numerical applications.

Tables II-V present solutions for each mode given in Table I, respectively. ADM solution procedure is conducted up to five iterations and a comparison with the numerical solutions of the same nonlinear differential equation is provided in each table. Both ADM and numerical solutions are obtained by Wolfram Mathematica Software. Solutions for each mode are obtained for different values of amplitude changing in the interval [0, 1] with a step size of 0.1. In Figures 1-4, absolute errors and total absolute errors are given for each mode. In the left part of the figures, absolute errors obtained from iterations of ADM are presented. As it can be seen, proposed technique introduces less error for fifth iteration solution. In the right part of the figures, cumulative absolute errors are given for all amplitudes.

Table I. Different values for parameters ε, σ and t

Mode	ε	σ	t
1	0.25	0.75	0.25
2	0.75	2.50	0.50
3	3.25	0.75	0.75
4	2.00	0.75	1.00

Table II. Comparisons for ADM solution and numerical solution for Mode 1

A	Numerical	$ADM_{n=1}$	$ADM_{n=2}$	$ADM_{n=3}$	$ADM_{n=4}$	$ADM_{n=5}$
0.1	0.098620	0.098406	0.098644	0.098618	0.098620	0.098620
0.2	0.197086	0.196625	0.197139	0.197081	0.197086	0.197086
0.3	0.295246	0.294469	0.295339	0.295237	0.295247	0.295246
0.5	0.490073	0.488281	0.490311	0.490047	0.490075	0.490072
0.7	0.682044	0.678344	0.682608	0.681973	0.682052	0.682043
0.9	0.870297	0.863156	0.871579	0.870105	0.870323	0.870294
1.0	0.962824	0.953125	0.964722	0.962511	0.962872	0.962817

Tables and Figures depict that absolute errors decrease when increasing numbers of iteration, i.e. the series in equation (15) is convergent and shows asymptotic behavior. Even with five iterations, ADM results show very good agreement with the numerical results. In Table VI, ADM results are compared with the previously available results (Ganji *et al.*, 2011; Ganji *et al.*, 2012). In Ganji *et al.*'s study (2012), differential transform method (DTM) was used to solve JTP and in Ganji *et al.*'s study (2011), homotopy perturbation method (HPM) and variational iteration method (VIM) were used to solve the same problem. ADM has a simpler algorithm providing an initial approximation with the formulation. The order of ADM approximation is chosen from the available order of solutions in the previous results.

Table VI depicts that ADM solutions show better agreement with the numerical results compared to VIM and DTM. In Ganji *et al.*'s study (2012), DTM approximation was conducted up to nine iterations. In Table VII, ninth-order solutions of ADM and DTM approximations and numerical solutions are compared. Since no solutions existed in Ganji *et al.*'s study (2012) for Mode 1, it was not included in the comparison of ninth-order

A	Numerical	$ADM_{n=1}$	$ADM_{n=2}$	$ADM_{n=3}$	$ADM_{n=4}$	$ADM_{n=5}$
0.1	0.087166	0.078000	0.091686	0.085448	0.087699	0.087025
0.2	0.174021	0.155250	0.183396	0.170401	0.175172	0.173705
0.3	0.260266	0.231000	0.275189	0.254350	0.262221	0.259695
0.5	0.429830	0.375000	0.459635	0.417019	0.434574	0.428194
0.7	0.593987	0.504000	0.647488	0.568275	0.605037	0.589361
0.9	0.751472	0.612000	0.843960	0.700680	0.777295	0.738293
1.0	0.827500	0.656250	0.948079	0.756328	0.866980	0.805290

Table III. Comparisons for ADM solution and numerical solution for Mode 2

A	Numerical	$ADM_{n=1}$	$ADM_{n=2}$	$ADM_{n=3}$	$ADM_{n=4}$	$ADM_{n=5}$
0.1	0.152310	0.170031	0.147326	0.153488	0.152071	0.152354
0.2	0.302013	0.338375	0.291890	0.304282	0.301578	0.302097
0.3	0.446731	0.503344	0.431061	0.449929	0.446203	0.446845
0.5	0.713915	0.816406	0.684158	0.718439	0.713895	0.713955
0.7	0.944676	1.095720	0.893222	0.951217	0.946858	0.943885
0.9	1.137780	1.327780	1.055400	1.151770	1.143340	1.133990
1.0	1.221360	1.421880	1.121950	1.243040	1.227550	1.215080

Table IV. Comparisons for ADM solution and numerical solution for Mode 3

A	Numerical	$ADM_{n=1}$	$ADM_{n=2}$	$ADM_{n=3}$	$ADM_{n=4}$	$ADM_{n=5}$
0.1	0.140700	0.162000	0.131930	0.143638	0.139875	0.140900
0.2	0.278053	0.321000	0.260198	0.283898	0.276498	0.278409
0.3	0.409103	0.474000	0.381490	0.417862	0.407003	0.409511
0.5	0.644004	0.750000	0.593750	0.659549	0.641495	0.643856
0.7	0.836798	0.966000	0.762510	0.863280	0.833060	0.835023
0.9	0.989290	1.098000	0.909570	1.027820	0.978607	0.988652
1.0	1.052570	1.125000	0.992188	1.087390	1.038810	1.055300

Table V. Comparisons for ADM solution and numerical solution for Mode 4 solutions. In Table VII, ADM results are shown in better agreement with the numerical solution especially for larger values of amplitudes.

In Figure 5, the variation of headway deviation with time is depicted and it is compared with the numerical solution. It can be seen that from figures, approximations are getting better with the increasing number of order. In Figure 5(a) and (b), fifth-order approximation is in very good agreement with the numerical solution up to half interval of time, whereas in Figure 5(c) and (d), fifth-order approximations are in very good agreement with the numerical solution for the whole interval of time. These behaviors are discussed in conclusion section.

6. Conclusion

In this study, ADM is used for the analysis of JTP in traffic congestion. The problem is modeled as a nonlinear non-conservative oscillator through Lorenz system. ADM is used for the first time in the solution of JTP. The problem was previously analyzed by using DTM, HPM and VIM. The proposed solution is obtained up to the ninth order of approximation and the results show that ADM provides good approximations even with less number of iterations. This behavior is due to a rapid convergence of the method even for a highly nonlinear differential equation governing the problem. Another advantage of the method is its simple formulation which already provides an initial approximation in the formulation

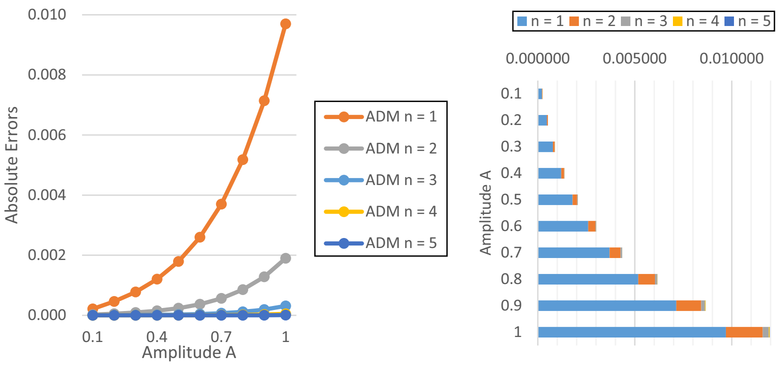

Figure 1.
The absolute errors and cumulative absolute errors for Mode 1

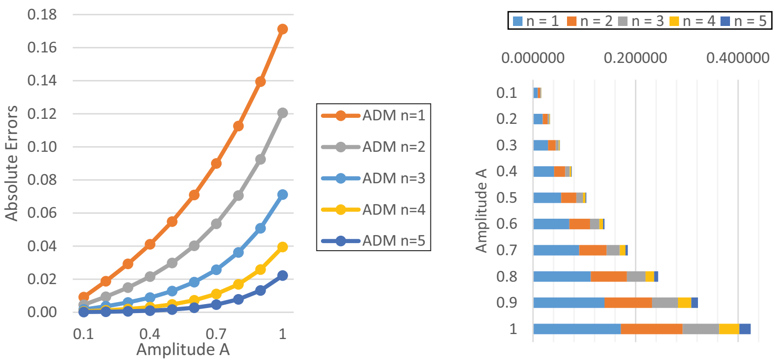

Figure 2.
The absolute errors and cumulative absolute errors for Mode 2

stage. ADM formulates the solution without any transformation of the equation, without any linearization procedure or without perturbation techniques. These properties of the method provide simplicity in the formulation when compared to other techniques.

The analyses showed that ADM with fifth-order approximation provided accurate results with negligible absolute errors. These solutions are in very good agreement with the numerical solution computed by the use of Mathematica software. Previous solutions to the problem obtained by using several methods were used for the comparison purposes. These methods are HPM, VIM and DTM. The results showed that ADM results are better approximations for all cases than VIM and DTM. ADM and HPM solutions with the same order of approximations were found to be very close or the same.

An initial headway deviation is assumed as an initial condition and fifth order of approximation is accurate enough for the solution given in Figure 5(c) and (d). However, additional terms to increase the order of approximation is required for better approximations, which may be observed in Figure 5(a) and (b). This behavior is due to the magnitude and the sign of $\varepsilon - \sigma$. Parameter σ is scaled inverse of relaxation time for velocity deviation and ε is scaled characteristic acceleration/breaking time. When $\varepsilon - \sigma$ is negative, headway deviation decreases with time and with the increase of negative magnitude, higher-order ADM approximations are required for an accurate solution. Negative sign indicates less characteristic acceleration/breaking time and this behavior leads to less congestion in traffic

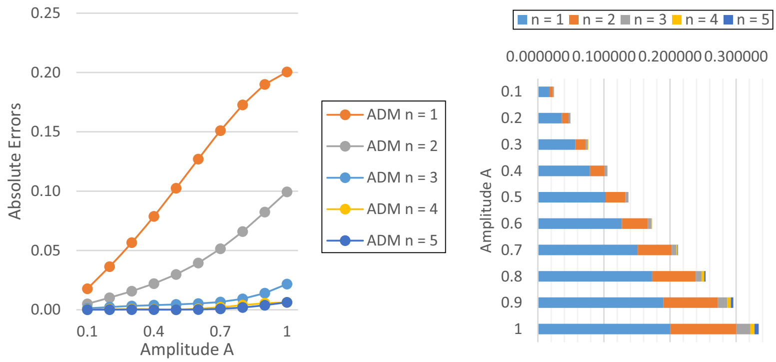

Figure 3. The absolute errors and cumulative absolute errors for Mode 3

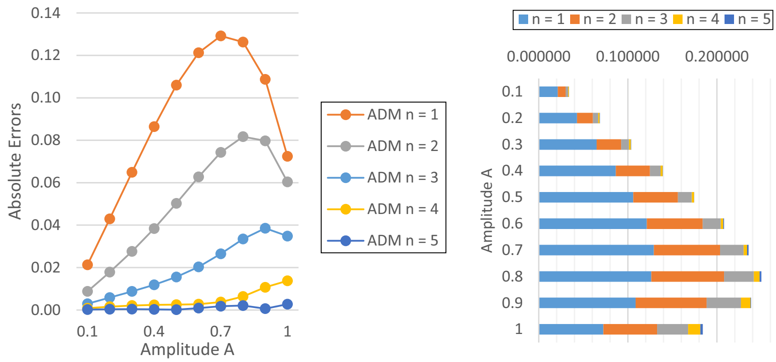

Figure 4. The absolute errors and cumulative absolute errors for Mode 4

Mode	A	Numerical	$ADM_{n=3}$	$DTM_{n=3}$	$HPM_{n=3}$	$VIM_{n=3}$
1	0.1	0.098620	0.098618	0.098406	0.098618	0.098618
	0.2	0.197086	0.197081	0.196620	0.197081	0.197081
	0.3	0.295246	0.295237	0.294470	0.295237	0.295237
	0.5	0.490073	0.490047	0.490230	0.490047	0.490049
	0.7	0.682044	0.681973	0.678340	0.681973	0.681981
	0.9	0.870297	0.870105	0.863160	0.870105	0.870135
	1.0	0.962824	0.962511	0.953120	0.962511	0.962570
2	0.1	0.087166	0.087699	0.090870	0.087699	0.085454
	0.2	0.174021	0.175172	0.181653	0.175172	0.170453
	0.3	0.260266	0.262221	0.272285	0.262221	0.245367
	0.5	0.429830	0.434574	0.453125	0.434574	0.418078
	0.7	0.593987	0.605037	0.634340	0.605037	0.572092
	0.9	0.751472	0.777295	0.818880	0.777294	0.711870
	1.0	0.827500	0.866980	0.914063	0.866980	0.774554
3	0.1	0.152310	0.152071	0.139218	0.152071	0.153550
	0.2	0.302013	0.301578	0.276455	0.301578	0.304792
	0.3	0.446731	0.446203	0.409806	0.446203	0.451683
	0.5	0.713915	0.713895	0.658203	0.713895	0.726723
	0.7	0.944676	0.946858	0.874116	0.946858	0.972742
	0.9	1.137780	1.143340	1.054001	1.143340	1.189564
	1.0	1.221360	1.227550	1.131836	1.227550	1.286738
4	0.1	0.140700	0.139875	0.125627	0.162000	0.131930
	0.2	0.278053	0.276498	0.248803	0.321000	0.260198
	0.3	0.409103	0.407003	0.367280	0.474000	0.381490
	0.5	0.644004	0.641495	0.583333	0.750000	0.593750
	0.7	0.836798	0.833060	0.767387	0.966000	0.762510
	0.9	0.989290	0.978607	0.929040	1.098000	0.909570
	1.0	1.052570	1.038810	1.010417	1.125000	0.992188

Table VI.
Comparisons for third-order results of ADM, DTM, HPM and VIM solutions

Mode	Method	A						
		0.1	0.2	0.3	0.5	0.7	0.9	1.0
2	ADM	0.087165	0.174019	0.260261	0.429796	0.593764	0.750126	0.827251
	DTM	0.087164	0.174018	0.260264	0.429839	0.594030	0.751465	0.827707
	Num.	0.087166	0.174021	0.260266	0.429830	0.593987	0.751472	0.827500
3	ADM	0.152310	0.302014	0.446734	0.713927	0.944709	1.137950	1.221760
	DTM	0.152354	0.302168	0.447132	0.715440	0.948418	1.144452	1.229225
	Num.	0.152310	0.302013	0.446731	0.713915	0.944676	1.137780	1.221360
4	ADM	0.140700	0.278058	0.409121	0.644096	0.837099	0.989567	1.052440
	DTM	0.140792	0.278331	0.409730	0.645857	0.839791	0.991466	1.053526
	Num.	0.140700	0.278053	0.409103	0.644004	0.836798	0.989290	1.052570

Table VII.
Comparisons for ADM and DTM ($n = 9$)

flow. When $\varepsilon - \sigma$ is positive, headway deviation increases with time and with the increase of positive magnitude, accurate results may be obtained with limited number of successive approximations that is equivalent to less order of ADM approximation. Positive sign indicates greater characteristic acceleration/breaking time and this behavior leads to more congestion in traffic flow due to greater characteristic acceleration/breaking time and the increasing headway deviation.

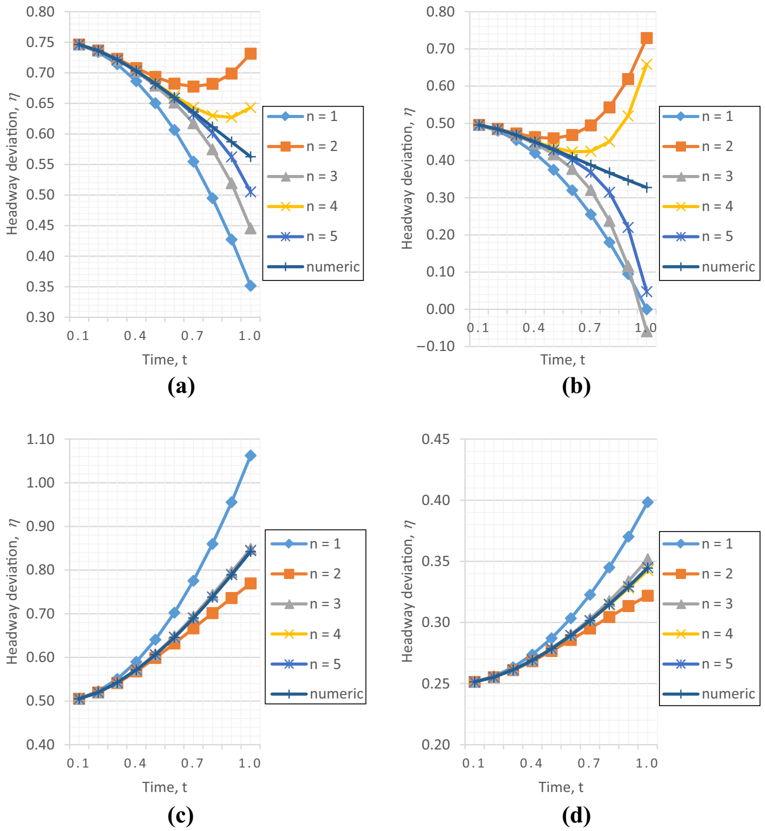

Figure 5. Variation of headway deviation with time (fifth order of approximation) and comparison with numerical solution

Notes: (a) η-t diagram with $\varepsilon = 0.25$ $\sigma = 0.75$ and A = 0.75; (b) η-t diagram with $\varepsilon = 0.75$, $\sigma = 2.50$ and A = 0.50; (c) η-t diagram with $\varepsilon = 3.25$, $\sigma = 0.75$ and A = 0.50; (d) η-t diagram with $\varepsilon = 2.00$, $\sigma = 0.75$ and A = 0.25

ADM showed a successful performance in the solution of JTP in traffic congestion and provided an analytical type solution of the problem with an easy formulation and application process.

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